3. The efficiency depends on the ambient condition (p_a and T_a)
4. High air rate is required to limit the maximum GT inlet temperature, as a result of which the exhaust losses are high, unless the waste heat in it is utilized
5. Compressor work required is quite large, which tells upon the efficiency of the plant
6. Air and gas filters have to be of very high quality so that no dust enters to erode and corrode the turbine blades

21.4 ANALYSIS OF A GT PLANT

The analysis of Brayton cycle, the air standard cycle of the GT plant, has been given in Chapter 13. The salient features of the cycle (Fig. 21.3) are being given below:

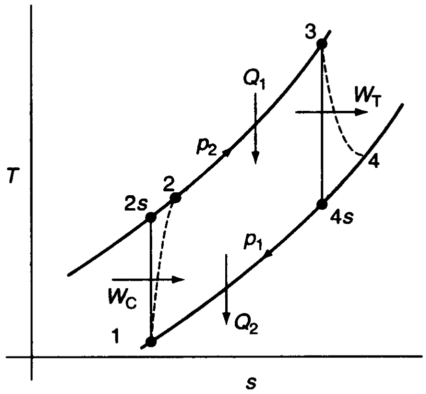


Fig. 21.3 Brayton cycle

Heat supplied, $Q_1 = m_a c_p (T_3 - T_2)$

Heat rejected, $Q_2 = m_a c_p (T_4 - T_1)$

$$\frac{T_{2s}}{T_1} = \frac{T_3}{T_{4s}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$$

where m_a = mass of air and r_p = pressure ration, p_2/p_1.

The compressor efficiency, $\eta_c = \dfrac{T_{2s} - T_1}{T_2 - T_1}$

and turbine efficiency, $\eta_T = \dfrac{T_3 - T_4}{T_3 - T_{4s}}$

For the ideal cycle, $1-2s-3-4s-1$,

$$\eta_{cycle} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \tag{21.1}$$

As r_p increases, η_{cycle} increases till Carnot cycle is reached (Fig. 21.4). With the increase or r_p, the mean temperature of heat addition T_{m_1} increases, and the mean temperature of heat rejection T_{m_2} decreases (Fig. 21.5). When in the limit $T_{m_1} \to T_3$ and $T_{m_2} \to T_1$, the Carnot efficiency is obtained.

$$\eta_{cycle} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{T_1}{T_3} = 1 - \frac{T_{min}}{T_{max}}$$

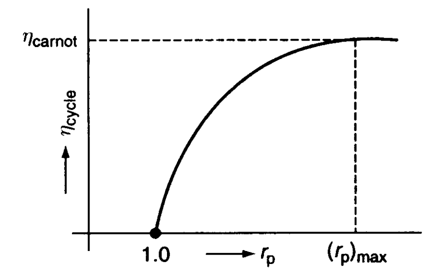


Fig. 21.4 Variation of η_{cycle} with r_p

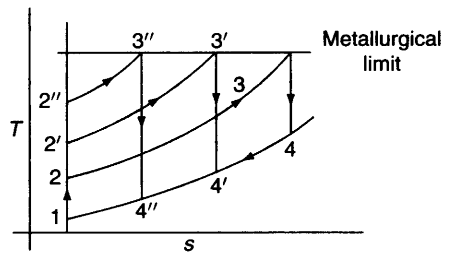


Fig. 21.5 Effect of r_p on Brayton cycle

$$\therefore \qquad (r_p)_{max} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (21.2)$$

For a particular value of r_p,

$$W_{net} = Q_1 - Q_2 = m_a c_p [T_3 - T_{2s} - T_{4s} + T_1]$$

$$= m_a c_p \left[T_3 - T_1 r_p^{\frac{\gamma-1}{\gamma}} - T_3 r_p^{-\frac{\gamma-1}{\gamma}} + T_1\right]$$

There is a particular value of r_p when W_{net} is maximum (Fig. 21.6). Now, the values of T_1 and T_3 are known.

Making $\frac{dW_{net}}{dr_p} = 0$, the optimum value of r_p becomes

$$(r_p)_{opt} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

Therefore, $(r_p)_{opt} = \sqrt{(r_p)_{max}}$

$$(W_{net})_{max} = m_a c_p \left[T_3 - 2\sqrt{T_3 T_1} + T_1\right]$$

$$= m_a c_p \left(\sqrt{T_3} - \sqrt{T_1}\right)^2 \qquad (21.3)$$

and

$$\eta_{cycle} = 1 - \sqrt{\frac{T_{min}}{T_{max}}} \qquad (21.4)$$

If the compressor and turbine efficiencies are considered, it can be shown

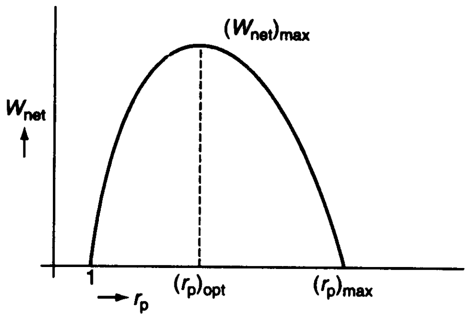
$$(r_p)_{opt} = \left(\eta_c \eta_T \frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} \qquad (21.5)$$

The work ratio r_w is defined as the ratio of net work to work done by the turbine.

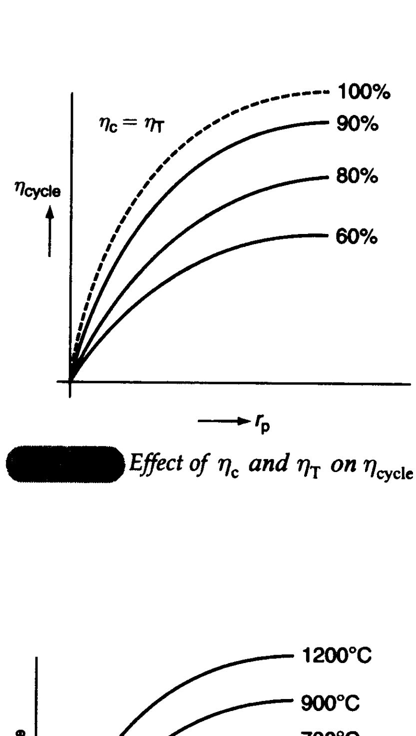
$$\therefore \qquad r_w = \frac{W_{net}}{W_T} = (W_T - W_c)/W_T = 1 - \frac{T_1}{T_3} r_p^{\frac{\gamma-1}{\gamma}} \qquad (21.6)$$

Figure 21.7 shows the effect of turbine and compressor efficiencies on cycle efficiency. The thermal efficiency of Brayton cycle or a GT plant is very sensitive to turbine inlet temperature T_3. As T_3 increases, the cycle efficiency increases (Fig. 21.8).

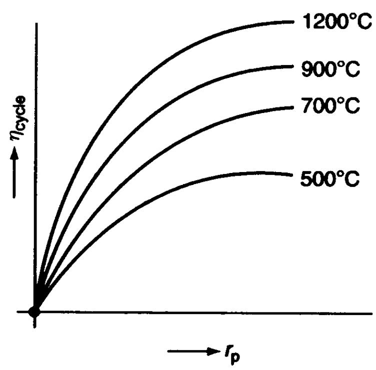
(a) Effect of Regeneration The thermal efficiency of a simple open-cycle gas turbine may be improved by the



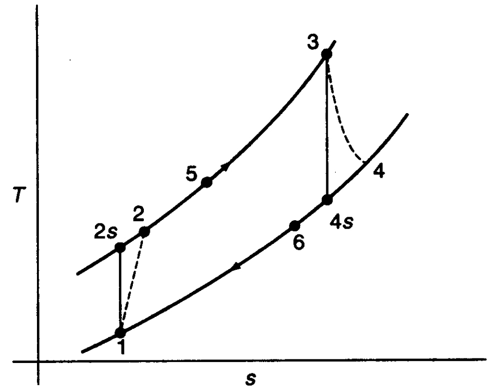
[redacted] *Effect of r_p on W_{net}*



[redacted] *Effect of η_c and η_T on η_{cycle}*



[redacted] *Effect of turbine inlet temperature on $\eta_{thermal}$ of a GT plant* utilization of the energy available in the turbine exhaust gases in a regeneration process. Inspection of Fig. 21.9 reveals that the temperature of the exhaust gases leaving the turbine at the state 4 is higher than the temperature of compressed air at the state 2. This difference in temperatures makes the regeneration possible. The recovery of a part of the thermal energy of the exhaust gases is accomplished by installing a heat exchanger called a *regenerator* in the flow system as shown in Fig. 21.10. The exhaust gases at a high temperature enter the hot side of the regenerator and are circulated around tubes containing the cold compressed air in the cold side of the regenerator. In this system, the temperature of the compressed air is increased before it reaches the combustion chamber (B) and therefore, less fuel is required to raise the air to the specified turbine inlet temperature. The effectiveness of regenerator is defined as

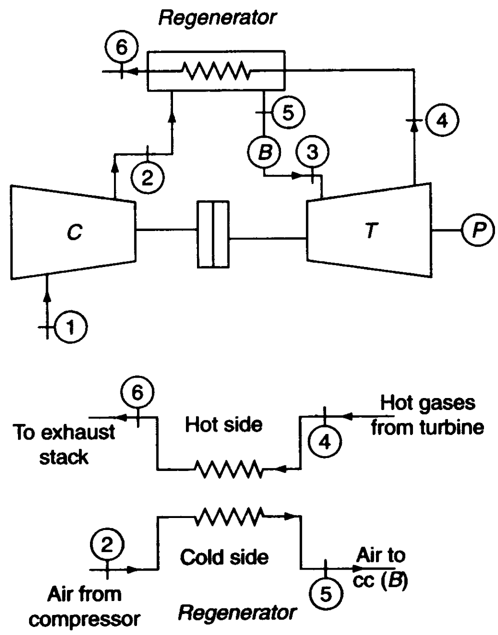


T-s diagram of an open-cycle gas turbine with regenerator

$$\varepsilon = \frac{\text{Actual temperature rise of air}}{\text{Maximum temperature rise possible}} = \frac{T_5 - T_2}{T_4 - T_2} \tag{21.7}$$

Here, $Q_1 = m_a c_p (T_3 - T_5)$ and $Q_2 = m_a c_p (T_6 - T_1)$, both of which decrease, whereas W_{net}, i.e., $(W_T - W_c)$ remains unchanged. Therefore, the efficiency of the cycle, W_{net}/Q_1, increases. In order to obtain an ideal regenerator, i.e., one having a heat exchanger effectiveness or regenerator efficiency of 100 per cent, the temperature of the compressed air, T_2, must be raised to the temperature of the exhaust gases, T_4, entering the regenerator. This could only be accomplished by having a heat-transfer surface of infinite area. Since the regenerators are restricted in size due to weight and space limitation, they have a maximum effectiveness of about 75 per cent.

(b) Effect of Intercooling By staging the compression process (1–2 and 3–4) with perfect intercooling (2–3), the cycle efficiency decreases, as shown in Fig. 21.11, where the small cycle 1–2–3–4–4′–1 is added to the basic cycle 1–4′–5–6–1 without intercooling. However, it permits more heat recovery from hot gases exiting the turbine at the state 6 by heating air leaving the compressor at the state 4. For minimum work of compression, the intercooler pressure $p_i = (p_1 p_2)^{1/2}$, where p_1 and p_2 are suction and discharge pressures, respectively.

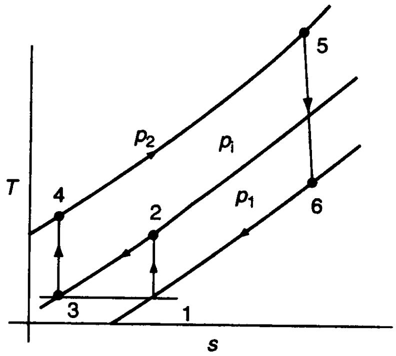


Open-cycle gas turbine with regenerator

(c) Effect of Reheating Similarly, by staging the heat supply process with a combustor and a reheater, the cycle efficiency decreases, but it permits more heat recovery from the turbine exhaust gases (Fig. 21.12) (since $T_6 > T_4'$) with the result that reheating along with regeneration may bring about an improvement in cycle efficiency.

It can be shown that the optimum reheat pressure for maximum net work output is

$$p_i = \sqrt{p_1\, p_2} \tag{21.8}$$



Effect of intercooling on Brayton cycle

(d) Effect of Intercooling, Reheating and Regeneration Figure 21.13(a) and (b) shows the flow and *T-s* diagrams of a closed-cycle GT plant with intercooling, reheating and regeneration.

The net work of the GT plant is given by

$$W_{\text{net}} = W_{\text{T}} - W_{\text{c}}$$

$$= (m_a + m_f)\, c_{p_g} \left[(T_6 - T_7) + (T_8 - T_9)\right]$$

$$- m_a c_{p_a} \left[(T_2 - T_1) + (T_4 - T_3)\right] \tag{21.9}$$

Heat supply,

$$Q_1 = (m_a + m_f)\, c_{p_g} \left[(T_6 - T_5) + (T_8 - T_7)\right]$$

$$= \dot{m}_f \times \text{C.V.}$$

Therefore, the overall plant efficiency

$$\eta_0 = \frac{W_{\text{net}}}{\dot{m}_f \times \text{C.V.}} \tag{21.10}$$

where C.V. = calorific value of the fuel.

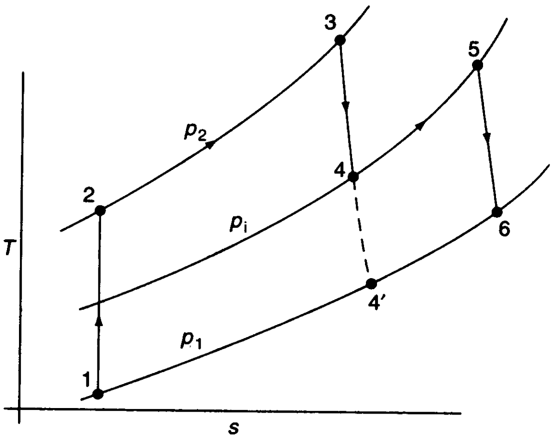


Fig. 21.12 Effect of reheat on Brayton cycle

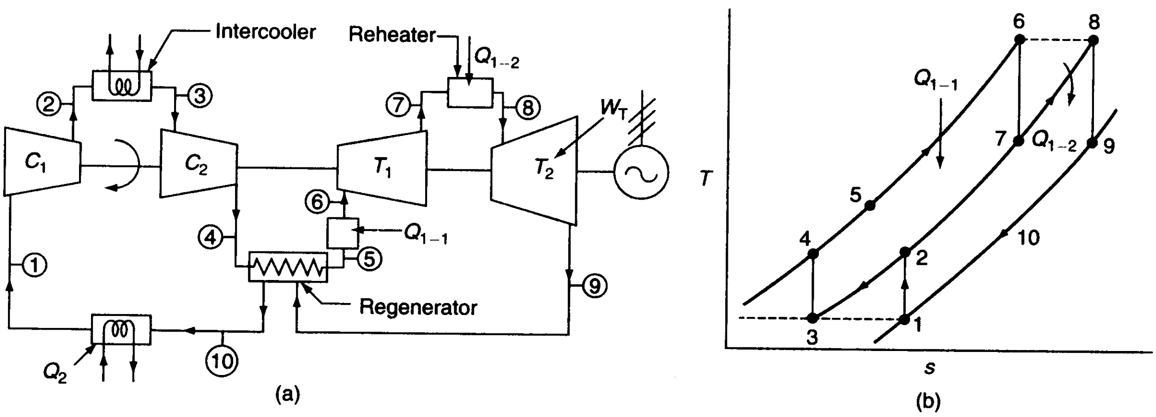
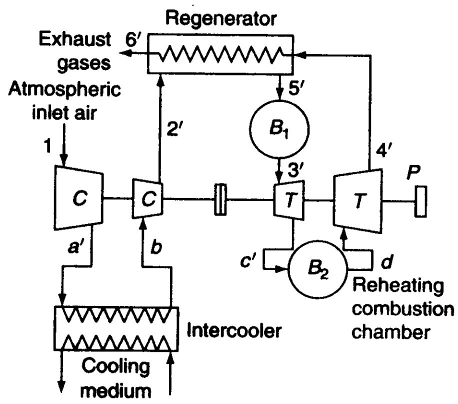


Fig. 21.13 Brayton cycle with intercooling, reheat and regeneration

Figure 21.14 shows an open-cycle gas turbine with a regenerator, intercooler and reheater.



Open-cycle gas turbine with regenerator, intercooler, and reheater

21.5 CLOSED-CYCLE GAS TURBINE

The engines discussed so far have been the open-cycle gas turbines. In a closed-cycle GT, the same working medium is continuously circulated (Fig. 21.2). The heat supply in the cycle takes place through a heat exchanger where a fuel may be burnt and heat rejection also occurs through a cooling medium in another heat exchanger. The performance characteristics, the effect of different variables and the component elements on the performance, and the equations developed for the open cycle apply equally as well to the closed cycle. The advantages of the closed cycle over that of the open cycle are (1) reduced size, (2) improved part-load efficiency (3) fuel flexibility. The disadvantages are (1) dependent system (cooling water availability) (2) complexity and cost, (3) air heater (not efficient in heat transfer).

21.6 SEMI-CLOSED CYCLE GT PLANT

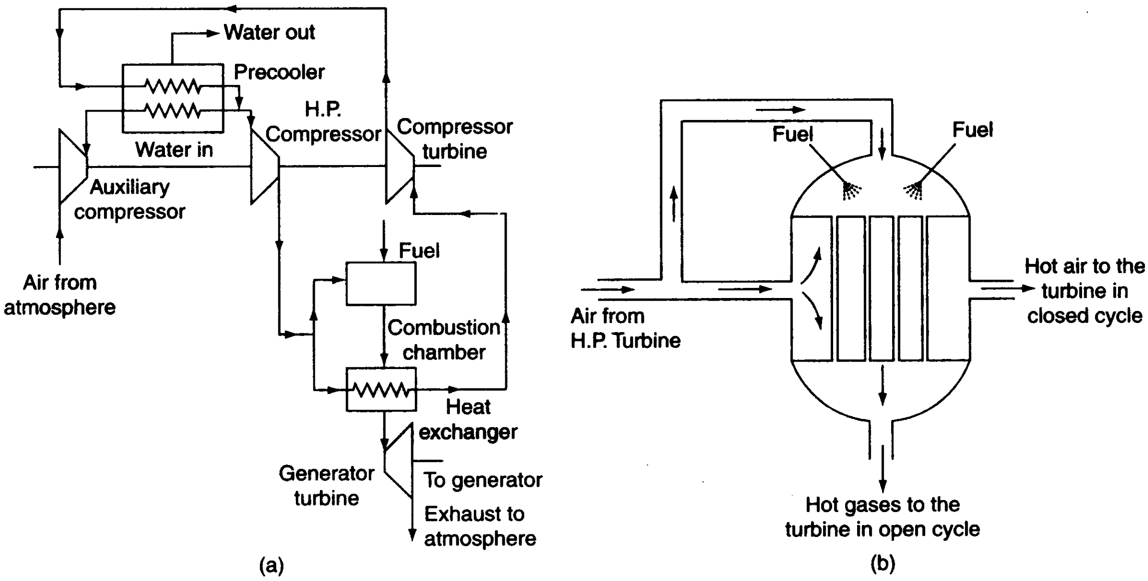
The advantages of the open cycle plant, viz. quick and easy starting and the closed cycle plant, viz. constant efficiency at all loads and higher unit rating permitting the use of higher back pressure, are combined in a semi-closed cycle gas turbine power plant. Here, part of the compressed air is heated by the gases exiting the combustion chamber (CC) and then expanded in an air turbine which drives the compressor, thus operating in a closed cycle. The remaining air is used in the CC to burn fuel, and the combustion products after heating the air expand in a gas turbine to drive the generator before exhausting to the atmosphere (Fig. 21.15 a). Figure 21.15 (b) shows a combined combustion chamber and a heat exchanger, where hot gases of combustion leave to expand in the gas turbine in the open cycle and the heated air flows to the air turbine in the closed cycle.

21.7 PERFORMANCE OF GAS TURBINE POWER PLANTS

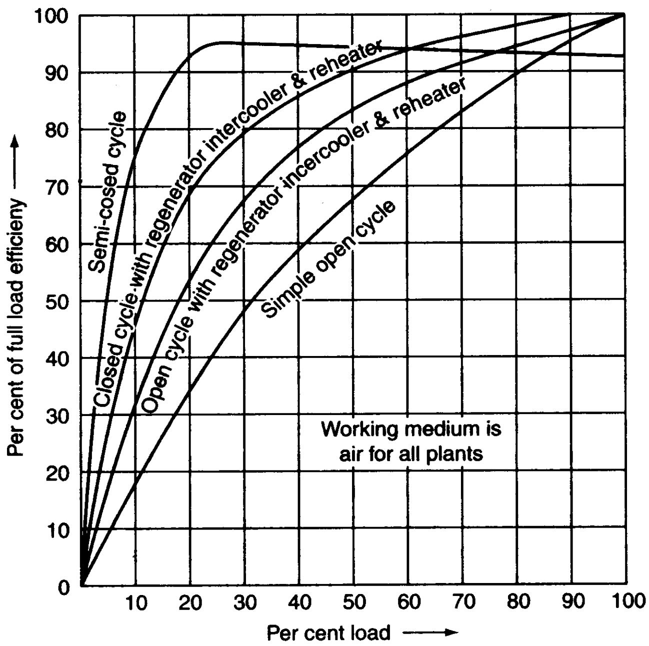
The gas turbine plant works under variable load conditions. It is thus necessary to study the effect of load on the cycle efficiency which is directly concerned with the running cost of the plant.

It is necessary to study the effect of pressure ratio on the thermal efficiency, air mass flow and specific fuel consumption with regenerative reheat and intercooled cycle, because smaller mass flow rate for the given output reduces the component sizes and the plant capital costs. Lower fuel consumption reduces the running cost of the plant. Some of these characteristics are represented graphically and also discussed.

(a) Part Load Efficiency The part load efficiencies for open cycle, closed cycle and semi-closed cycle are shown in Fig. 21.16. The part load performance of the semi-closed cycle is seen to be the best.



(a) Semi-closed cycle gas turbine plant, (b) Combined combustion chamber and air heater



Part load efficiencies of different plants

(b) *Fuel Consumption* The effect of pressure ratio on the specific fuel consumption (sfc) of an open cycle plant with the degree of regeneration as a parameter is shown in Fig. 21.17. It shows that for each degree of regeneration there is an optimum pressure ratio for minimum sfc.

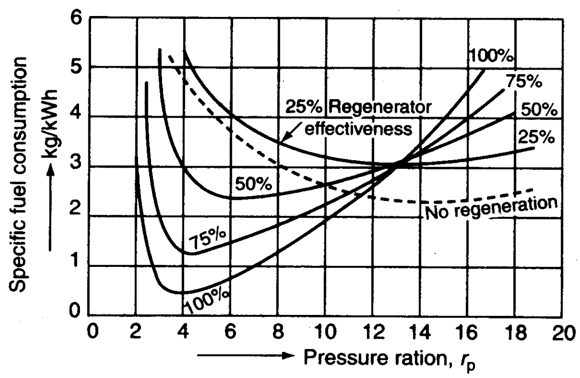
(c) *Air Rate* In the addition to the thermal efficiency which is a measure of the fuel economy, the size of the plant is equally important in many applications, particularly in the field of aviation. For a given duty, the size of a plant is dependent on the air flow rate in relationship to the useful shaft output. The air rate is defined as

$$\text{AR} = \frac{w_a\ \text{kg/s} \times 3600}{W_{net}\ (\text{kW})}, \quad \text{i.e. kg/kWh.}$$

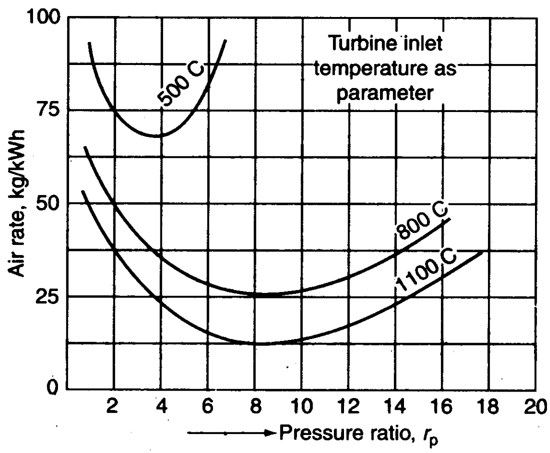
Air rate is a criterion of the size of the plant, i.e., the lower the air rate, the smaller the plant. From the mechanical and metallurgical standpoint, the lowering of the air rate results in turbines of smaller physical dimensions with a more uniform temperature distribution. Any means by which the physical dimensions can be reduced and the inherent distortions minimized are steps toward greater reliability of the gas turbine . The effect of pressure ratio on air rate for an open cycle plant with the turbine inlet temperature as a parameter is shown in Fig. 21.18. It indicates optimum pressure ratio for different turbine inlet temperatures requiring minimum air rates.

(d) *Thermal Efficiency* The effect of pressure ratio of a simple open-cycle plant with turbine inlet temperature as a parameter is shown in Fig. 21.19 and with compressor inlet temperature as a parameter in Fig. 21.20. As the turbine inlet temperature increases for a particular pressure ratio, thermal efficiency increases, and for each temperature, there is an optimum value of r_p when efficiency is maximum. An increase in the compressor inlet air temperature increases the compressor work. W_{net} is decreased, air rate increases and thermal efficiency decreases.

(e) *Regeneration* The effect of regeneration on thermal efficiency of a simple cycle, taking pressure ratio and turbine inlet air temperature as parameters, is shown in Fig. 21.20(a) and Fig. 21.20(b) respectively.



Effect of regenerator effectiveness on specific fuel consumption



Effect of pressure ratio on air mass flow per unit output

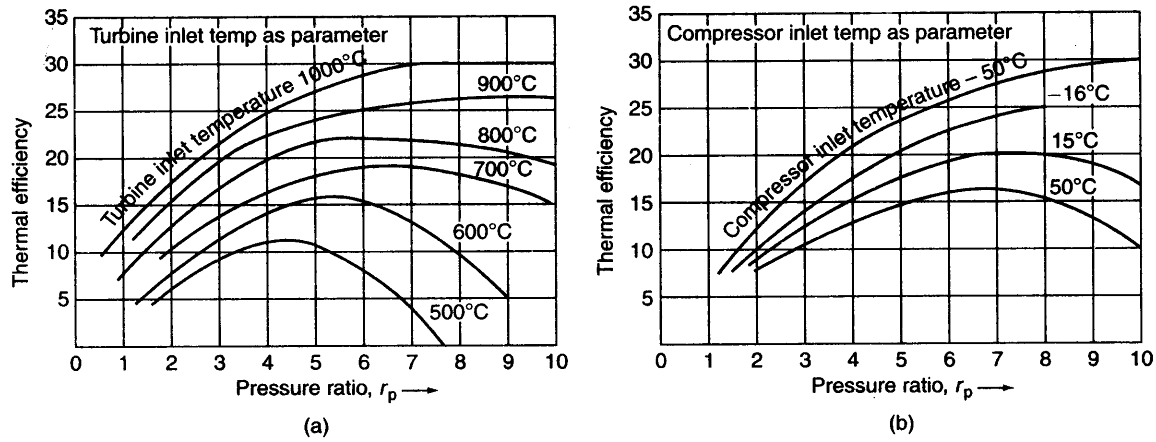


Fig. 21.19 *Effect of pressure ratio on thermal efficiency of a simple open cycle plant with (a) Turbine inlet temperature and (b) Compressor inlet temperature as parameters*

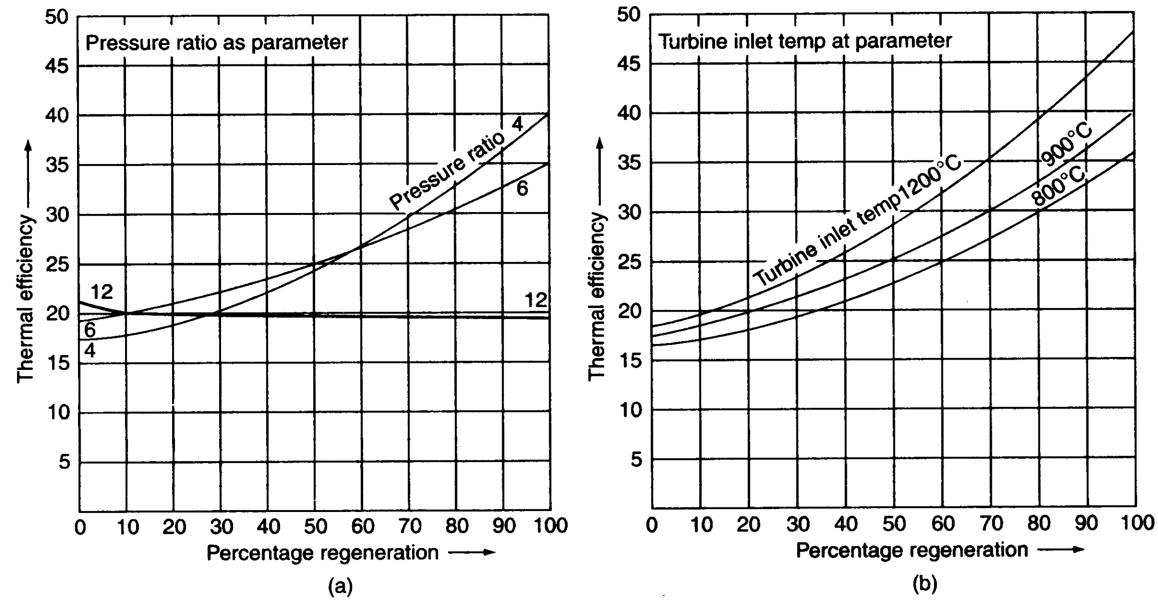


Fig. 21.20 *Effect of regeneration on thermal efficiency of simple cycle with (a) Pressure ratio and (b) Turbine inlet temperature as parameters*

21.8 COMPONENTS OF A GAS TURBINE PLANT

The construction and operation of the components of a gas turbine plant are necessary for proper understanding and design.

(a) Compressor The high flow rates of air through the turbine and the relatively moderate pressure ratios necessiate the use of rotary compressors. The types of compressors commonly used are

1. Centrifugal compressors
2. Axial flow compressors

These have been discussed in Chapter 19. The centrifugal compressor is comprised of two major parts, the impeller, or rotating component and the diffusor. The air enters the compressor at the hub and it then moves radially outward through the impeller and into the diffusor. The impeller converts the mechanical energy, available to the compressor, into kinetic energy, plus heat due to friction, in the working media. The diffusor then transforms the kinetic energy in the air into pressure energy in accordance with Bernoulli's principle. The flow through the diffusor is subject to frictional losses as well. Also, because the air leaves the impeller radially, it must normally be turned 90° to enter the combustion chamber or regenerator, involving more frictional losses. The choice of the blade shape (i.e., bent backward, forward or straight radialers) and the compressor rpm depend on stress limits and manufacturing costs.

In general, the *centrifugal compressor*, as compared to axial flow, is more rugged, simpler, relatively insensitive to surface deposits, has a wider stability range, is less expensive, and attains a higher pressure ratio per stage. However, the efficiency is lower, the diameter larger, and it is not readily adaptable to multi-staging. The single-stage compressors for use in industry may obtain efficiencies from 80 to 84% at pressure ratios between 2.5 and 3, while for aircraft use, pressure ratios are between 4 and 4.5 with efficiencies in the range of 76 to 81%.

The important characteristics of the *axial flow compressor* are its high peak efficiencies, adaptability to multistaging to obtain higher overall pressure ratios, high flow-rate capabilities, and relatively small diameter. However, the axial flow compressor is sensitive to changes in air flow and rpm, which result in a rapid drop off in efficiencies, i.e., the stability range of speeds for good efficiencies is small.

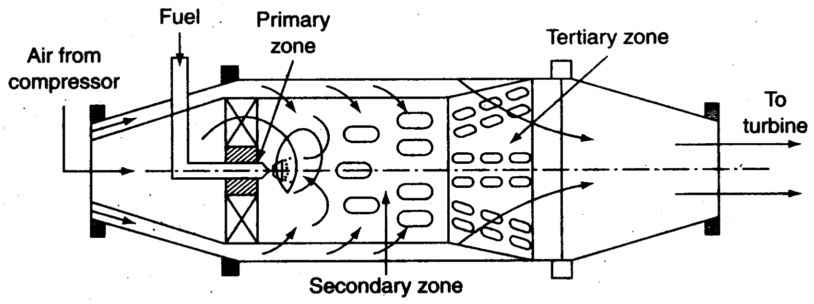
The axial flow compressor consists of a series of rotor-stator stages. The rotor comprises a series of blades that move relative to a series of stationary blades called the stator. The blades transmit the mechanical energy into kinetic energy in the air. Compression is accomplished in both the rotor and stator blades into pressure energy (i.e., continually diffusing it from a high velocity to a lower velocity with a corresponding rise in static pressure). The details of the flow diagram and the velocity triangles as well as the power input and efficiency of the compressor are given in Chapter 19.

(b) Combustion Chamber (CC) Its characteristic feature is that combustion of fuel has to take place with high air velocities (50 to 140 m/s) to limit the size of the cc and with high air fuel ratios (50:1 to 250:1) to keep the turbine inlet temperature to permissible limits.

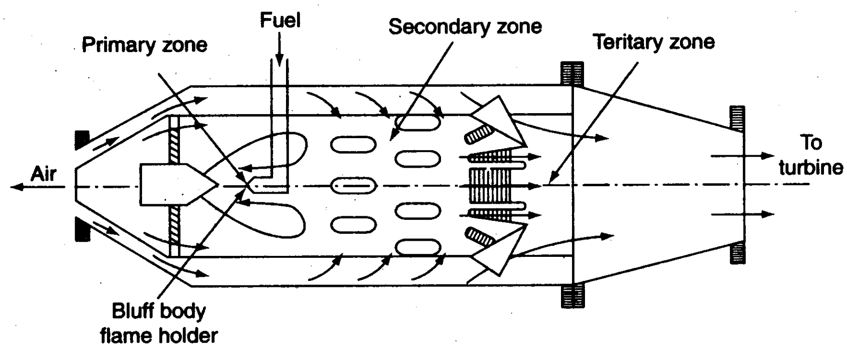
In an open cycle GT plant combustion may be arranged to take place in one or two large cylindrical can-type combustion chambers (CC) with ducting to convey the hot gases to the turbine. Combustion is initiated by an electric spark and once the fuel starts burning, the flame is required to be stabilized. A pilot or recirculated zone is created in the main flow to establish a stable flame which helps to sustain combustion continuously. The common methods of flame stabilization are by swirl flow and by bluff body.

Figure 21.21 shows a can-type combustor with swirl flow flame stabilization. About 20 per cent of the total air from the compressor is directly fed through a swirler to the burner as primary air, to provide a rich fuel-air mixture in the primary zone, which continuously burns, producing high temperature gases. Air flowing through the swirler produces a vortex motion creating a low pressure zone along the axis of the CC to cause reversal of flow. About 30 per cent of total air is supplied through dilution holes in the secondary zone through the annulus round the flame tube to complete the combustion. The secondary air must be admitted at right points in the CC, otherwise the cold injected air may chill the flame locally thereby reducing he rate of reaction. The secondary air not only helps to complete the combustion process but also helps to cool the flame tube. The remaining 50 per cent of air is mixed with burnt gases in the tertiary zone to cool the gases down to the temperature suited to the turbine blade materials.

Figure 21.22 shows a can-type combustor with a bluff body stabilizing the flame. The fuel is injected upstream into the air flow and a sheet metal cone and perforated baffle plate ensure the necessary mixing of



Can-type combustor with swirl flow flame stabilizer



Can-type combustor with bluff-body flame stabilizer

fuel and air. The low pressure zone created downstream side causes the reversal of flow along the axis of the CC to stabilize the flame. Sufficient turbulence is produced in all three zones of the CC for uniform mixing and good combustion.

The air-fuel ratio in a GT plant varies from 60/1 to 120/1 and the air velocity at entry to the CC is usually not more than 75 m/s. There is a rich and a weak limit of flame stability and the limit is usually taken at flame blowout. Instability of the flame results in rough running with consequent effect on the life of the CC.

Because of the high air-fuel ratio used, the gases entering the HP turbine contain a high percentage of oxygen and therefore if reheating is performed, the additional fuel can be burned satisfactorily in HP turbine exhaust, without needing further air for oxygen.

A team "combustion efficiency" is often used in this regard, which is defined as follows.

$$\text{Combustion efficiency} = \frac{\text{Theoretical fuel–air ratio for actual temperature rise}}{\text{Actual fuel air ratio for actual temperature rise}}$$

Theoretical temperature rise depends on the calorific value of the fuel used, the fuel-air ratio and the initial temperature of air. To evaluate the combustion efficiency, the inlet and outlet temperatures and the fuel and air mass flow rates are measured. The fuel used in aircraft gas turbine is a light petroleum distillate or kerosene of gross calorific value of 46.4 MJ/kg. For gas turbines used in power production or in cogeneration plants, the fuel used can be natural gas.

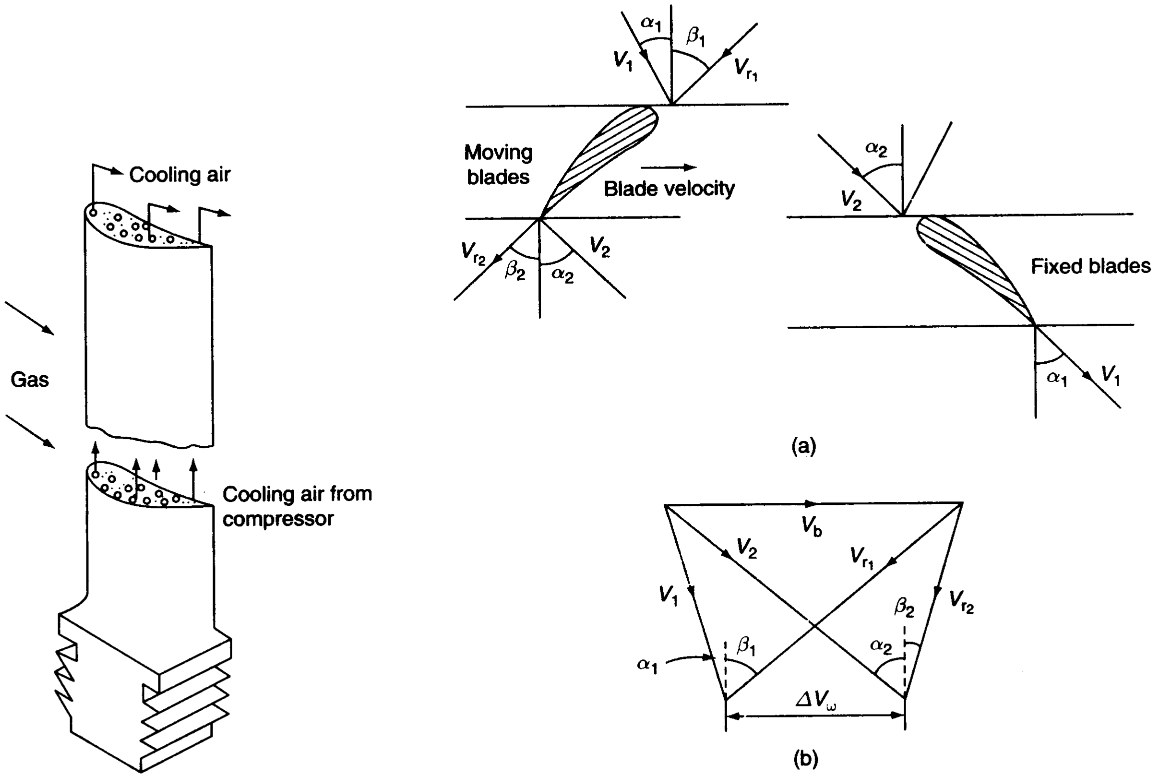
In order to give a comparison of combustion chambers operating under different ambient conditions, a combustion intensity is defined as the following.

$$\text{Combustion intensity} = \frac{\text{Heat release rate}}{\text{Volume of CC} \times \text{inlet pressure}}$$

The lower the combustion intensity, the better the design. In aircraft a figure of about 23 kW/(m^{3}atm) is normal, whereas in large industrial plants it is about 0.2 kW/(m^3 atm).

The performance criteria of the combustion chamber are (1) low pressure loss, (2) high combustion efficiency, and (3) good flame stability. Flame stability implies steady and continuous flame. This is serious to the phenomena of resonant or pulsating combustion, and due to blowouts, where the flame is blown out of the exit of the CC and is thereby extinguished, which may happen in aircraft applications. The main requirements for a good CC are (1) low carbon deposit in the CC, turbine, and regenerator, (2) low weight and frontal area, (3) reliability and serviceability, and (4) thorough mixing of cold air with the hot products of combustion to give uniform temperature distribution.

(c) Gas Turbines Like steam turbines, gas turbines are also of the axial-flow type (Fig. 21.23). The basic requirements of the turbines are light weight, high efficiency, reliability in operation and long working life. Large work output can be obtained per stage with high blade speeds when the blades are designed to sustain higher stresses. More stages are always preferred in gas turbine power plants, because it helps to reduce the stresses in the blades and increases the overall life of the turbine. The cooling of gas turbine blades is essential for long life as it is continuously subjected to high temperature gases.



Typical air cooled gas turbine blade. (a) Typical blade sections (b) Blade velocity diagrams for an axial-flow compressor

Blade angles of gas turbines follow the axial-flow compressor blading (Fig. 21.23(a)), where the degree of reaction is not 50 per cent. It is usually assumed for any stage that the absolute velocity at inlet to each stage (V_2) is equal to the absolute velocity at exit from the moving blades (i.e. V_2) and that the same flow velocity V_f is constant throughout the turbine.

The degree of reaction, R, as defined for a steam turbine, is valid for gas turbines also. It is the ratio of the enthalpy drop in the moving blades to the enthalpy drop in the stage. As shown in Fig. 21.23(a), we have

$$R = \frac{V_{r_2}^2 - V_{r_1}^2}{2V_b \Delta V_w} = \frac{V_f^2(\sec^2\beta_2 - \sec^2\beta_1)}{2V_b(V_f\tan\beta_2 + V_f\tan\beta_1)}$$

$$= \frac{V_f(\tan^2\beta_2 - \tan^2\beta_1)}{2V_b(\tan\beta_2 + \tan\beta_1)} = \frac{V_f}{2V_b}(\tan\beta_2 - \tan\beta_1) \tag{21.11}$$

Putting $R = 0.5$ in Eq. (21.11), we get $\quad V_f(\tan\beta_2 - \tan\beta_1) = V_b$

or $\quad V_b + V_f\tan\alpha_2 - V_f\tan\beta_1 = V_b \quad \alpha_2 = \beta_1$

It also follows that $\alpha_1 = \beta_2$. The fixed and moving blades have the same cross-section and the diagram is symmetrical.

Vortex Blading is the name given to the twisted blades which are designed by using three dimensional flow equations with a view to decrease fluid flow losses. A radial equilibrium equation can be derived (*see the book of Cohen et al.*) and it can be shown that one set of conditions which satisfies this equation is as follows.

(a) Constant axial velocity along the blades, i.e. V_f = constant.

(b) Constant specific work over the annulus, i.e. $V_b\Delta V_w$ = constant.

(c) Free vortex at entry to the moving blades. i.e. $V_{w1}r$ = constant, where r is the blade radius at any point.

Since the specific work output is constant over the annulus, it can be calculated at the mean radius, and multiplied by the mass flow rate it becomes the power for the stage. Since the fluid density varies along the blade height, the density at the mean radius can be used, so that $\dot{m} = \rho_m V_f A$, where A is the blade annular area.

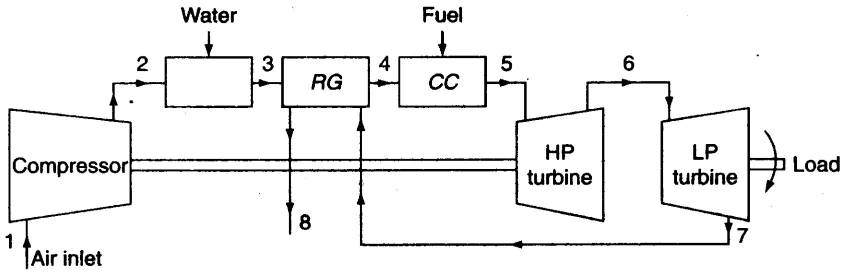
(d) *Duct work* The duct work consists of ducts between the compressor and the combustion chamber, combustion chamber to the turbine, and the exhaust duct. The ducts must be sized to minimize the pressure losses, as the loss in pressure directly reduces the capacity of the plant.

Ducts should be supported from the floor to reduce vibrations. Expansion joints must be provided to allow for dimensional changes due to temperature variation.

The basic requirements for the turbines are light weight, high efficiency, ability to operate at high temperatures for long periods, reliability and serviceability. The determination of blading material depends on the stress-rupture and creep characteristics of the various blading materials, in combination with mechanical and thermal stresses, resistance to mechanical and thermal shock, and resistance to corrosion and vibration.

21.9 GAS TURBINE WITH WATER INJECTION

One method used to improve the performance of the gas turbine is to inject water into the air between the compressor and the regenerator (Fig. 21.24). The quantity of water injected is just sufficient to saturate the compressed air. Any excess water injected may cause fouling of the regenerator. Increased mass flow rate flowing through the turbine increases the turbine output without increasing the compressor work input. Water



Schematic layout of a two-shaft open cycle gas turbine with water injection and regenerator

injection is most commonly used as a power boost for take-off and emergency requirement with jet propelled aircraft (see later). However, if water has impurities, it may cause corrosion or deposits on the blades, which will have a detrimental effect on the performance and maintenance.

21.10 GAS TURBINE FUELS

Gas turbines are basically designed to operate on petroleum-based fuels like natural gas, kerosene, aviation fuel and residual fuel oil. Other fuels like powdered coal, sewage gas, etc. are also being actively considered.

(a) Natural Gas It generally contains a major percentage of methane and a small percentage of ethane propane and butane. The sulphur compound (H_2S) is kept below 0.1 per cent by volume. It is obtained from wells in oil-fields and used for auxiliary power production within the oil-fields.

(b) Liquid Fuels The liquid fuels range from gas oil to residual fuel oil. The major consideration in selecting the fuel is cost. The cheapest fuels are always preferred.

Distillate fuels in the gas oil range (boiling point varies between 200°C to 370°C) may be used in a gas turbine without any difficulty. Residual fuels include fuel oils, furnace oils, boiler fuel oils, etc. If the viscosity of oil is high, some heating arrangement need be provided. Caution should be observed against corrosion of blades and other components by sulphur compounds and vanadium.

(c) Solid Fuels The use of coal as fuel for closed cycle gas turbine plant is universally accepted, but its use in open cycle plant is now in active development. Coal is burnt in two modes, viz. (i) integrated gasification, where coal is completely or partially gasified and the fuel gas produced is consumed in the gas turbine combustor and (ii) pressurized bubbling or circulating fluidized bed, where the fuel gas, after it is adequately filtered, expands in the gas turbine. Coal is normally considered as a gas turbine fuel in combined cycle power generation. The development of proper filters, ceramic or others, is the key to its use.

21.11 GAS TURBINE MATERIALS

The combustion chamber of a diesel engine is subjected to the highest temperature of the cycle, 2000°C, to 2500°C, for a very short period during only one stroke of the cycle. During the remaining three strokes, the engine gets time to be cooled. Therefore, special materials are not required for diesel engine plants.

In gas turbine plants, however, the components are continuously exposed to the hot gases and are made of special materials with necessary arrangements for cooling. Blades are also subjected to high centrifugal stresses due to high rotative speeds, in addition to thermal stress. These have a high creep rate, due to which the blades increase in length gradually. Contact with the casing can thus occur with resultant failure. Blade materials should possess the following properties.

(i) Materials must withstand high temperature and high stress.
(ii) It must have low creep rate.
(iii) It must have high resistance to oxidation, corrosion and erosion.
(iv) It must neither be brittle at ordinary temperature nor plastic like when hot.
(v) It must have good castability or forgeability characteristics, depending on the process of manufacture.
(vi) It must have good machinability to achieve precise dimension.
(vii) It must have high resistance to fatigue failure.
(viii) It must maintain structural stability when exposed to varying temperature.

All these required properties cannot be obtained in one material. Therefore, the selection of material for each component is a difficult job.

1. *Metals for Turbine Rotor Discs* The turbine rotor disc is subjected to centrifugal and thermal stresses. The thermal stresses (due to temperature gradient) can be reduced by using an alloy of high conductivity.

The disc hub stresses tend to cause tensile deformation. This can be minimised by using a material of low expansion coefficient. Austenitic steels with 12 to 18 per cent chromium, 8 to 12 per cent nickel and small percentages of tungsten, molybdenum and titanium are used for turbine rotor discs.

These days the turbine discs are cooled by tapping compressed air from the compressor. Therefore, less expensive materials can be used. Ferritic steels having higher creep strength at low temperature (up to 600°C) can be used for the central portion, whereas austenitic steel is used on the outer surface of the ferritic rotor disc.

2. *Material for Turbine Rotor Blade* Blades are subjected to the highest stresses and temperatures. Most satisfactory materials for blades are the stainless steel alloys and 8-20 nickel chromium alloys, known as Nimonic alloys. These alloys have high resistance to oxidation, scaling with ceramics (silicon carbide, silicon nitride, aluminium nitride, etc.) on the blades of nimonic alloys provides better mechanical properties. Blades are cooled by compressed air taken by a bleed from the compressor.

3. *Material for Combustion Chamber* The gas turbine combustion chamber is generally made of Nimonic 75 alloy. This alloy has an excellent creep resistance, capacity to withstand heavy thermal shocks, and high resistance to oxidation.

4. *Material for Compressor* The impeller of centrifugal compressor is subjected to high centrifugal and thermal stresses, the latter being due to the temperature difference between the air inlet and air discharge temperatures. To minimise centrifugal stresses, lighter materials like aluminium alloys are used. These alloys suffer from high thermal expansion, for which allowance is provided.

The axial flow compressor blades are now made of titanium alloys, which are of low density, possess good strength at high temperatures (400–500°C) and are strongly resistant to corrosion. Light weight, good creep strength and fatigue resistance are attractive features of titanium alloys.

21.12 JET PROPULSION SYSTEM

Jet propulsion, like all means of propulsion, is based on Newton's second and third laws of motion. Newton's second law states that *the rate of change of momentum in any direction is proportional to the force acting in that direction*. Newton's third law states that *for every action there is an equal and opposite reaction*.

With regard to vehicles operating entirely in a fluid, the reaction principle is based on imparting momentum to a mass of fluid in such a manner that the reaction of the imparted momentum furnishes a propulsive force. Peculiar to jet propulsion, however, this mass of fluid, whose velocity has been increased, is rejected from the vehicle in a jet stream. The jet aircraft draws in air and expels it to the rear at a markedly increased velocity; the rocket greatly changes the velocity of the fuel which it ejects rearward in the form of products of combustion. In each case, the *action* of accelerating the mass of fluid in a given direction creats a *reaction* in the opposite direction in the form of a propulsive force. The magnitude of this propulsive force is defined as *thrust*.

Aircraft propulsion may be achieved by using a heat engine to drive an airscrew or propeller, or by allowing a high-energy fluid to expand and leave the aircraft in a rearward direction as a high-velocity jet. In the propeller type of aircraft engine, the propeller takes a large mass flow and gives it a moderate velocity backwards relative to the aircraft. In the jet engine, the aircraft induces a relatively small air flow and gives it a high velocity backwards relative to the aircraft. In both cases the rate of change of momentum of the air provides a reactive forward thrust which propels the aircraft. The propeller-type engine can be driven by a petrol engine or by a gas turbine unit.

If the velocity of the jet backwards relative to the aircraft is V_j and the velocity of the aircraft is V_o, then the atmospheric air, initially at rest, is given a velocity of $(V_j - V_o)$ (Fig. 21.25). The thrust available for propulsion is solely due to the rate of change of momentum of the air stream.

$$\text{Thrust per unit mass-flow rate} = V_j - V_o$$

$$\text{propulsive power is then} = V_o(V_j - V_o)$$

This is the rate at which work must be done in order to keep the aircraft moving at the constant velocity V_o against the frictional resistance or drag.

$$\text{Net work output from the engine} = \text{Increase in kinetic energy of air} = V_j^2 - V_o^2$$

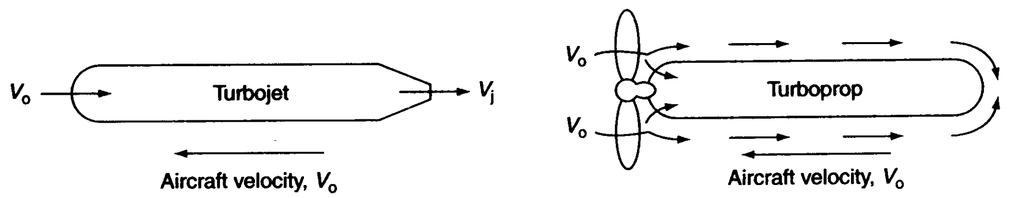
It is used in the two ways: (i) it provides the thrust and (ii) it gives the air, previously at rest, an absolute velocity $(V_j - V_o)$ and KE $(V_j - V_o)^2/2$. Therefore,

$$V_o(V_j - V_o) + \frac{(V_j - V_o)^2}{2} = V_o V_j - V_o^2 + \frac{V_j^2 - 2V_jV_o + V_o^2}{2}$$

$$= \frac{V_j^2 - V_o^2}{2}$$

$\therefore$ Propulsive efficiency,

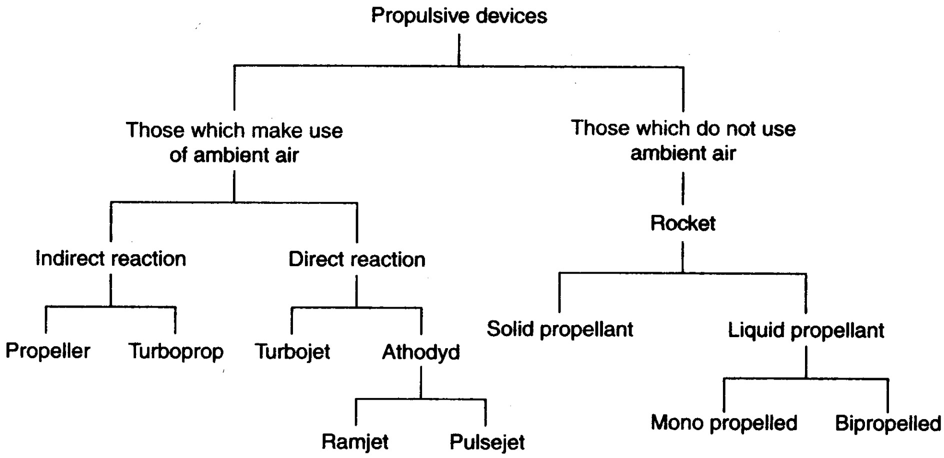
$$\eta_p = \frac{2V_o(V_j - V_o)}{V_j^2 - V_o^2} = \frac{2V_o}{V_j + V_o}$$



Flow through a turbojet and turboprop

21.13 PROPULSIVE DEVICES FOR AIRCRAFTS AND MISSILES

Various propulsive devices used for aircrafts and missiles are given below:



Propulsive devices are basically of two types: those which make use of atmospheric air as the main working medium supplying oxygen for combustion of fuel, *the atmospheric jet engines*, and those which carry oxygen required for combustion of fuel, the *rockets*. The performance of jet engines depends on the forward speed of the engine and upon the atmospheric pressure and temperature. The rocket engine carries its own oxidizer for the combustion of fuel and is, therefore, independent of the atmospheric air as well as the forward speed.

The devices which make use of the ambient air are further subdivided into indirect reaction and direct reaction devices. The main propulsive devices are the following:

1. *Propeller* It is an indirect reaction device. Earlier, it used to be driven by the reciprocating internal combustion engine. A propeller handles relatively a large mass of air and accelerates it rearwards at low speeds. It is the reaction of the rate of change of momentum of the air, called the thrust, which propels the aircraft. The function of the engine is only to revolve the propeller at the desired speed. Piston engines are, however, now used only for small aircrafts.

2. *Turbojet* A turboject is the most important direct reaction device. It utilizes a gas turbine power plant. In a turbine, partial expansion takes place to produce just sufficient power to drive the compressor. The exhaust of the turbine which is at a pressure higher than the atmospheric pressure is expanded in a nozzle given a high-velocity jet. Compared to propeller units, in turbojet units a small mass of air flows through the unit, but has a high rearward velocity. Turbojets are very efficient at high speed and high altitude, and inefficient at low speed and low altitude.

3. *Turboprop* It is a combination of indirect and direct reaction devices (propeller and turbojet). Thrust is produced both by propeller and jet. Besides the compressor the turbine also drives the propeller through a reduction gear. It has the thermal advantage of a turbojet, combined with the advantages of the propeller for efficient take-off, particularly for the heavily loaded aircraft.

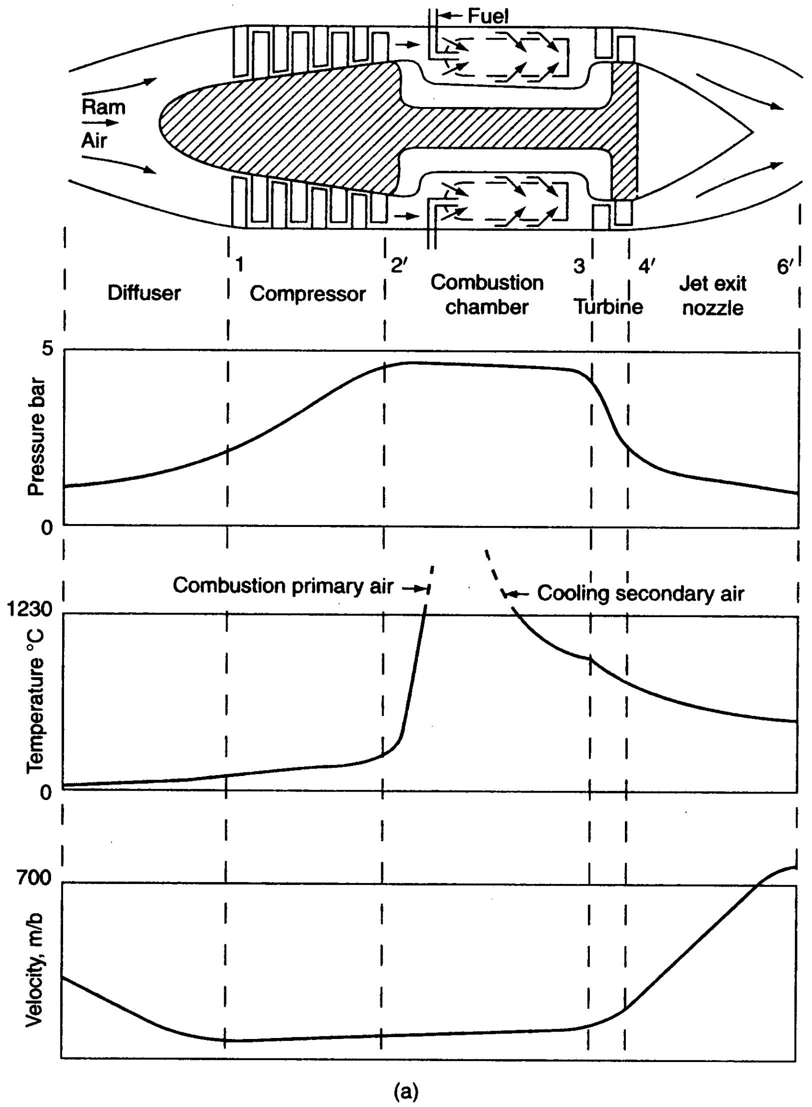
4. *Athodyd (Aero-thermodynamic Duct)* The ram jet and the pulse jet are athodyds, i.e., a straight duct-type of jet engine without a compressor and turbine wheels. The entire compression is obtained by a ram, eliminating the need of a turbine. Athodyds are used for pilotless aircraft, helicopter rotor and missiles.

5. *Rocket* It does not use ambient air for propulsion. Both the fuel and oxidizer are carried with the power plant, and are accelerated from zero velocity to a high velocity at nozzle outlet. A rocket is the only propulsion device suitable for space travel.

21.13.1 Turbojet Engine

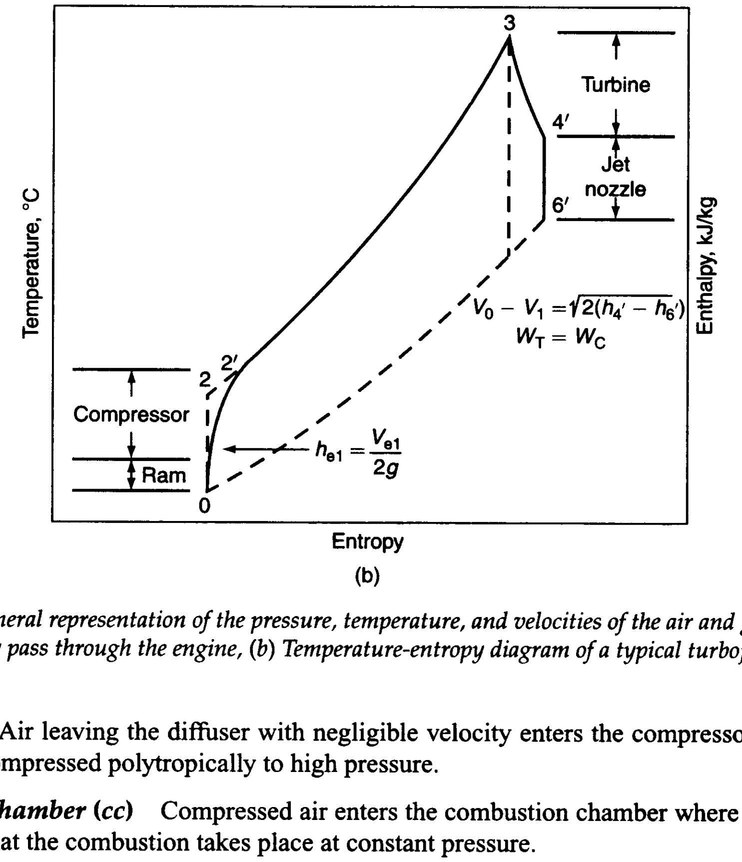
The basic components of a turbojet engine are shown in Fig. 21.26 are:

(a) Diffuser The ambient air enters with a velocity equal to the velocity of the aircraft and this velocity is slowed down in the diffuser. The kinetic energy of the airstream is converted to pressure energy. This is called 'ram compression'.



(a)

Fig. 21.26 Continued



(b)

Fig. 21.26 *(a) General representation of the pressure, temperature, and velocities of the air and gases of combustion as they pass through the engine, (b) Temperature-entropy diagram of a typical turbojet engine cycle*

(b) Compressor Air leaving the diffuser with negligible velocity enters the compressor (usually an axial flow type) and is compressed polytropically to high pressure.

(c) Combustion Chamber (cc) Compressed air enters the combustion chamber where the fuel is sprayed and it is assumed that the combustion takes place at constant pressure.

(d) Turbine The products of combustion of high pressure and temperature undergo polytropic expansion to an intermediate pressure such that the output of the turbine is just sufficient to run the compressor and the auxiliaries of the unit.

(e) Nozzle The gases coming out of the turbine expand down to the ambient pressure and a high-velocity jet leaves the nozzle. This produces the required thrust and the aircract is propelled in the forward direction. This method of propulsion is best suited for aircrafts flying with a speed of 800 km/h or more.

The temperature – entropy diagram for a typical turbojet engine is given in Fig. 21.26(b). The entering atmospheric air is diffused isentropically from velocity V_0 down to zero ($V_1 = 0$) in process 0–1. The hot gases leaving the turbine are assumed to expand isentropically (process 4′–6′), and the turbine work W_T is equal to the compressor work ($W_T = W_c$). The rise in ram pressure ratio p_1/p_0 increases with Mach number. At a Mach number of 2, the ideal ram-pressure ratio is 8. The thrust T is the magnitude of propulsive force created by the jet engine depending on the rate of change of air flowing through the engine. Since the weight rate of flow of fuel through the engine is normally in the vicinity of 1% of the rate of flow of air, it will not introduce any appreciable error if it is assumed that the working medium is comprised of air only. It can be expressed as

$$T = w_a(V_j - V_o) \text{ newtons}$$

where w_a = mass flow rate of air, kg/s; V_j = exit velocity of gases leaving the nozzle, m/s; and V_o = vehicle velocity through the air, m/s. Since the atmospheric air is assumed to be at rest, the velocity of air entering the engine is the velocity of the vehicle V_o.

Thrust power, TP, the time rate of development of the useful work achieved by the engine, is the product of the thrust times the flight velocity of the vehicle, or

$$\mathrm{TP} = TV_o = w_a(V_j - V_o)\,V_o \tag{21.12}$$

Propulsive power, PP, representing the energy required to change the momentum of the mass flow of air, may be expressed as the difference between the kinetic energies of the entering air and the exit gases, or

$$\mathrm{PP} = \frac{w_a\,(V_j^2 - V_o^2)}{2} \tag{21.13}$$

Therefore, the *propulsive efficiency*, η_p, may be expressed as

$$\eta_p = \frac{\mathrm{TP}}{\mathrm{pp}} = \frac{2(V_j - V_o)V_o}{V_j^2 - V_o^2} = \frac{2V_o}{V_j + V_o} = \frac{2}{1 + \dfrac{V_j}{V_o}} \tag{21.14}$$

This is also often called the Froude efficiency. As $V_o \to V_j$, η_p approaches maximum value. But as this occurs, the thrust and propulsive power approach zero. Thus, the ratio of velocities (V_j/V_o) for maximum efficiency and for maximum power are not the same.

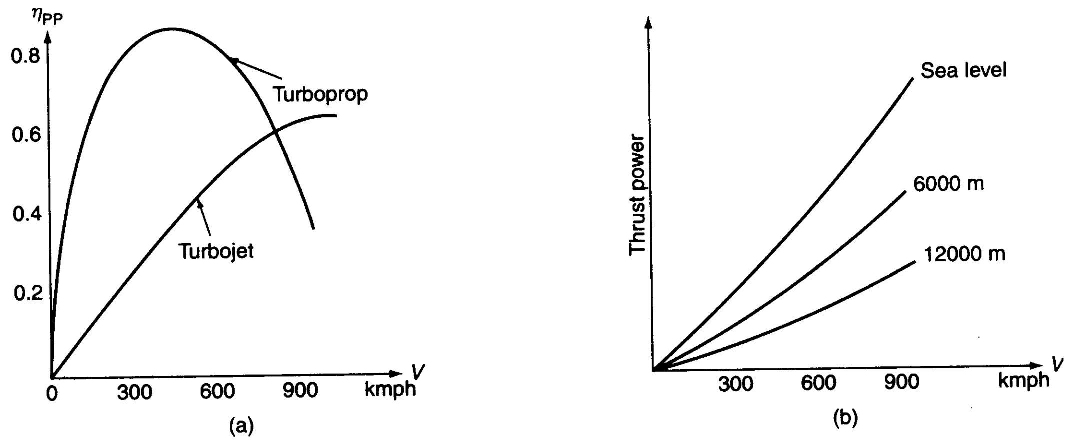
An alternate way of defining propulsive efficiency is to express propulsive power in terms of thrust power and kinetic energy losses, i.e.,

$$\mathrm{PP} = \mathrm{TP} + \text{K.E. losses.}$$

The propulsive efficiency then becomes

$$\eta_p = \frac{\mathrm{TP}}{\mathrm{PP}} = \frac{\mathrm{TP}}{\mathrm{TP} + \text{K.E. losses}} \tag{21.15}$$

The main components of a turbojet unit and of on open-cycle gas turbine unit are the same and as such, the performance of a turbojet unit depends on the pressure ratio of the compressor, efficiency of the individual constituents and the turbine inlet temperature. Since the turbojet unit propels an aircraft at different altitudes with varying speed, the performance of these units is a function of the flight speed and altitude. Figure 21.27(a) shows the variation of propulsive efficiency with flight speed and Fig. 21.27(b) shows the variation of thrust power with flight speed at different altitudes. It is seen that as the aircraft velocity



(a) Variation of the propulsive efficiency with flight speed, (b) Variation of thrust power with flight velocity at different altitude for a turbojet engine

V_o increases, the propulsive efficiency increases. For a propeller driven aircraft the change of η_p is greater initially, but at speeds at which the propeller tip attains the sonic velocity, η_p falls off rapidly and Eq. (21.14) is no longer valid. For aircraft speeds upto about 850 km/h, the propeller is the more efficient means of propulsion, but for speeds above this the jet engine is superior.

The thrust power decreases with increasing altitude because the density of the ambient air decreases with increasing altitude, But it has been found that the drag on the aircraft decreases at a greater rate than the air density and therefore, it is possible to fly the aircraft at a higher speed and with better economy.

21.13.2 Pressure Thrust

In aircraft gas turbine work it becomes important to use stagnation conditions, since velocity changes through the unit is no longer negligible. Also, in general, temperature - measuring instruments such as thermocouples, measure stagnation temperature and not static temperature: using stagnation conditions, the isentropic efficiencies of the compressor and turbine, and intake duct and nozzle or jet pipe efficiency are redifined.

Referring to Fig. 21.28(b) of a typical jet engine, we have

$$(\eta_s)_{\text{intake duct(ram)}} = \frac{T_{01s} - T_0}{T_{01} - T_0}, \qquad (\eta_s)_{\text{compressor}} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$(\eta_s)_{\text{turbine}} = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}, \qquad (\eta_s)_{\text{nozzle}} = \frac{T_{04} - T_{05}}{T_{04} - T_{5s}}$$

For adiabatic flow, the total temperature remains constant and $T_0 = T_{01}$. There is a loss of pressure in the combustion chamber from 2 to 3.

It was assumed that the gases expand down to atmospheric pressure in the jet nozzle. In the case of a convergent nozzle, the back pressure will normally be lower than the nozzle exit pressure. This phenomenon is called *underexpansion*, which is explained in Chapter 17.

Due to the difference in pressure between the nozzle exit and the atmosphere in which the aircraft is flying, there will be an additional thrust, called the *pressure thrust*. In the case of supersonic aircraft, the pressure at the air intake is higher than the atmospheric pressure because of compression through the shock wave formed, which will reduce the net thrust calculated purely from momentum considerations.

If we consider an aircraft like the turbojet in Fig. 21.28(a) with an air intake of area A_1, inlet air pressure p_1, and a nozzle exit area A_2, exit pressure p_2, and the atmospheric pressure p_0, we have from Newton's second law of motion,

$$F + p_1 A_1 - p_2 A_2 = \text{rate of change of air in the direction of motion of the fluid}$$

where F is the net force due to the hydrostatic pressure and friction exerted by the inside of the aircraft on the working fluid in the direction of its motion,

$$\therefore \quad F + p_1 A_1 - p_2 A_2 = \dot{m}(V_j - V_o)$$

$$\therefore \quad F = \dot{m}(V_j - V_a) - p_1 A_1 + p_2 A_2.$$

There is an equal and opposite force, R; exerted by the working fluid on the inside of the aircraft engine,

$$R = \dot{m}(V_j - V_a) - p_1 A_1 + p_2 A_2$$

in the direction of motion of the aircraft.

Let us consider the forces acting on the aircraft. There is the force R, there is the total drag D due to the air resistance, and there is a pressure force due to the atmospheric pressure acting on the projected area in the direction of flight. In flight there is considerable pressure variation over the aircraft surfaces causing lift and drag forces, the total drag force D and the form drag due to the vortices formed.

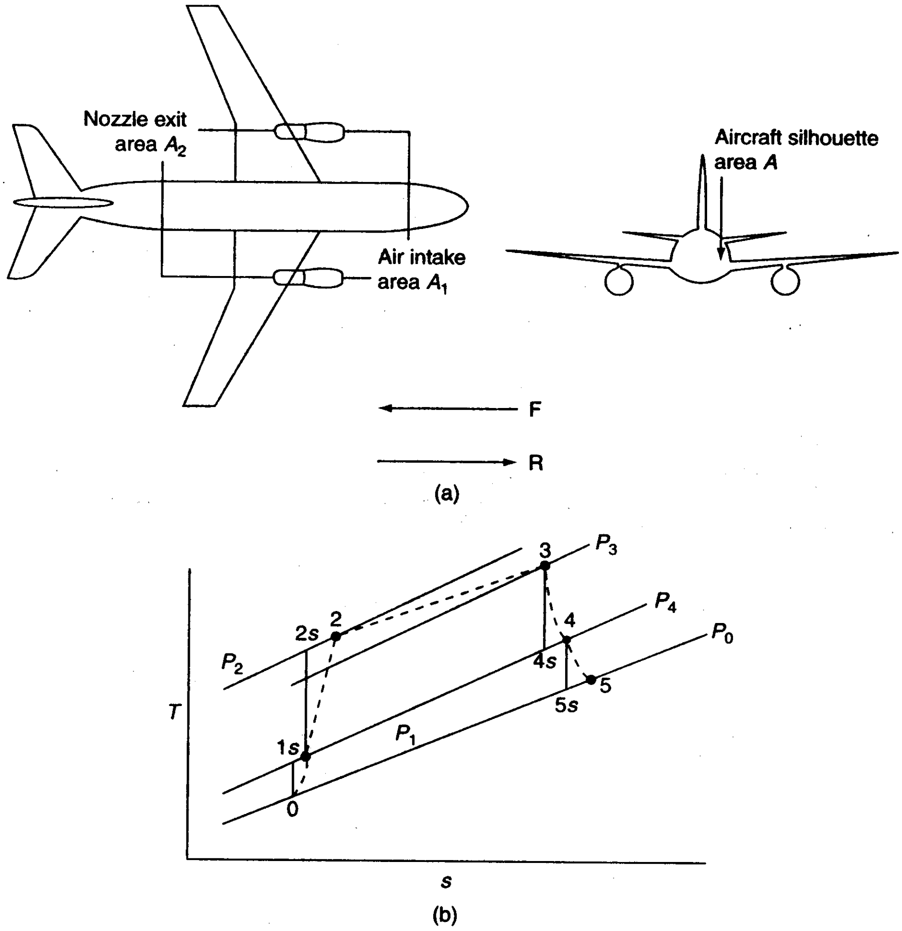


Fig. 21.28 *(a) Plan view of aircraft and aircraft silhouette (b) T–s diagram of a jet engine*

Assuming that the aircraft silhouette (profile) area in the direction of flight is A, then the net pressure force in the direction of flight is given by

$$p_0(A - A_2) - p_0(A - A_1) = p_0(A_1 - A_2).$$

Since the aircraft is flying at constant velocity, the net force acting is zero, i.e,

$$R - D + p_0(A_1 - A_2) = 0$$

Therefore, the total thrust required to overcome the total drag force is given by

$$\text{Total thrust } (T) = D = R + p_0\,(A_1 - A_2)$$

$$= \dot{m}(V_j - V_o) - p_1 A_1 + p_2 A_2 + p_0(A_1 - A_2)$$

or, $$T = \dot{m}(V_j - V_o) + A_2(p_2 - p_0) - A_1(p_1 - p_0)$$

For subsonic aircraft, however, the last term is zero, since then $p_1 = p_o$.

$\therefore$ $$\text{Total thrust } = \dot{m}\left(V_j - V_0\right) + A_2(p_2 - p_0) = \text{Momentum thrust} + \text{Pressure thrust}.$$

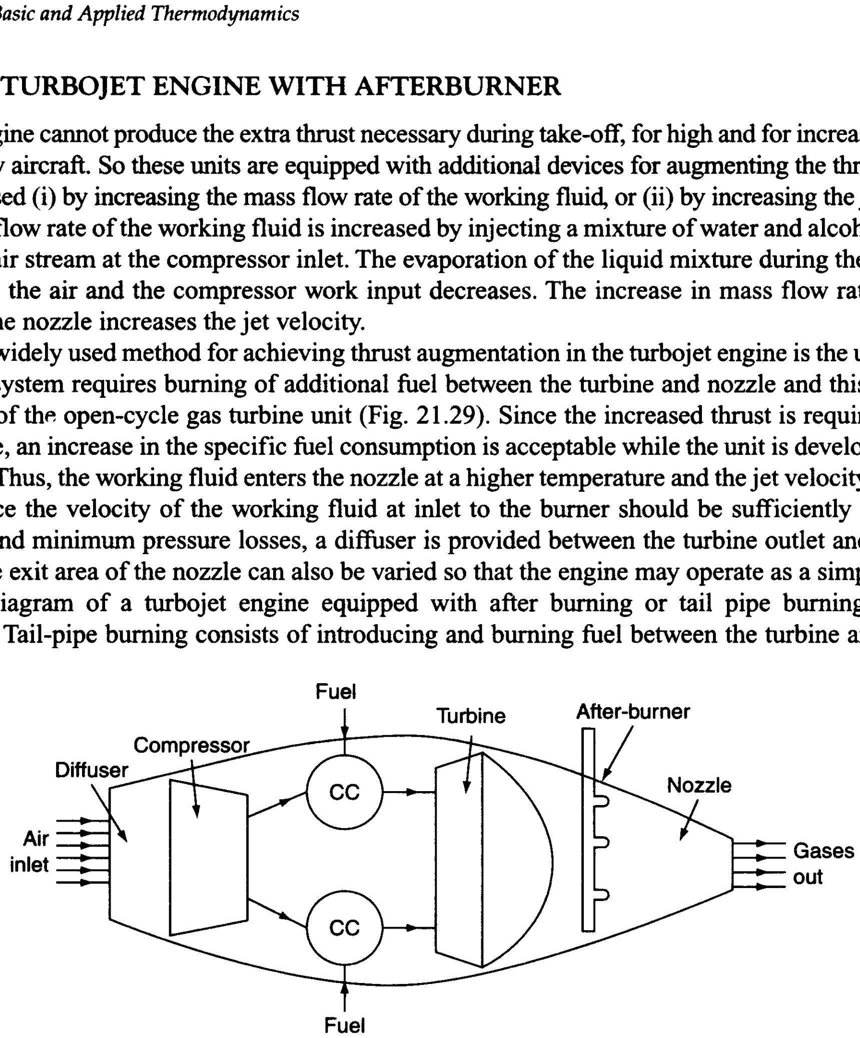
21.14 TURBOJET ENGINE WITH AFTERBURNER

A turbojet engine cannot produce the extra thrust necessary during take-off, for high and for increased maneuvering of military aircraft. So these units are equipped with additional devices for augmenting the thrust. The thrust can be increased (i) by increasing the mass flow rate of the working fluid, or (ii) by increasing the jet velocity.

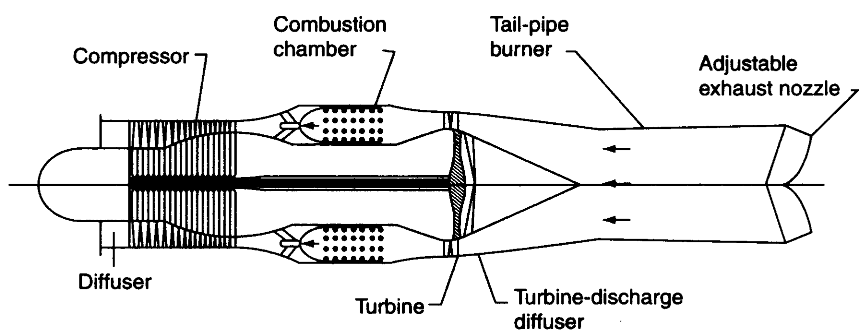
The mass flow rate of the working fluid is increased by injecting a mixture of water and alcohol (methanol) into the ram air stream at the compressor inlet. The evaporation of the liquid mixture during the compression process cools the air and the compressor work input decreases. The increase in mass flow rate through the turbine and the nozzle increases the jet velocity.

The most widely used method for achieving thrust augmentation in the turbojet engine is the use of an afterburner. This system requires burning of additional fuel between the turbine and nozzle and this is equivalent to a reheater of the open-cycle gas turbine unit (Fig. 21.29). Since the increased thrust is required for a short period of time, an increase in the specific fuel consumption is acceptable while the unit is developing the additional thrust. Thus, the working fluid enters the nozzle at a higher temperature and the jet velocity at exit would increase. Since the velocity of the working fluid at inlet to the burner should be sufficiently low for stable combustion and minimum pressure losses, a diffuser is provided between the turbine outlet and burner inlet. Moreover, the exit area of the nozzle can also be varied so that the engine may operate as a simple turbojet.

Another diagram of a turbojet engine equipped with after burning or tail pipe burning is shown in Figure 21.30. Tail-pipe burning consists of introducing and burning fuel between the turbine and the nozzle,



Schematic layout of a turbojet engine with an after-burner

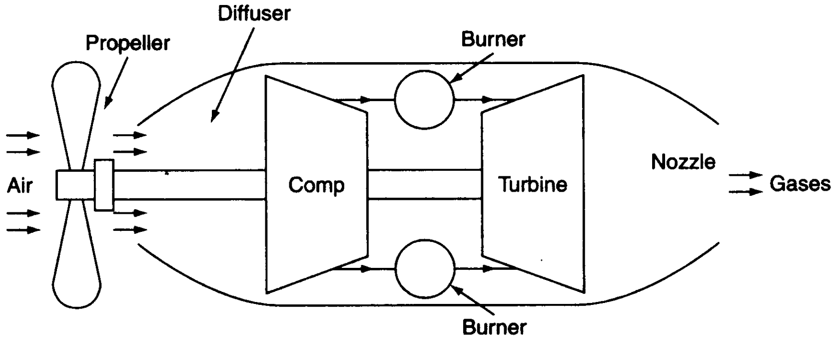


Turbojet engine with afterburner or with tail-pipe burning raising the gas exit velocity and producing an increased thrust. Since the turbojet with tail-pipe burning consists of a turbojet and essentially a ram jet(explained later), the engine is sometimes designated a turbo-ram-jet. Tail-pipe burning is not only an augmentation device for improving the take-off and high speed performance of an airplane, but it also may be considered a distinct type of engine for flight at supersonic speed.

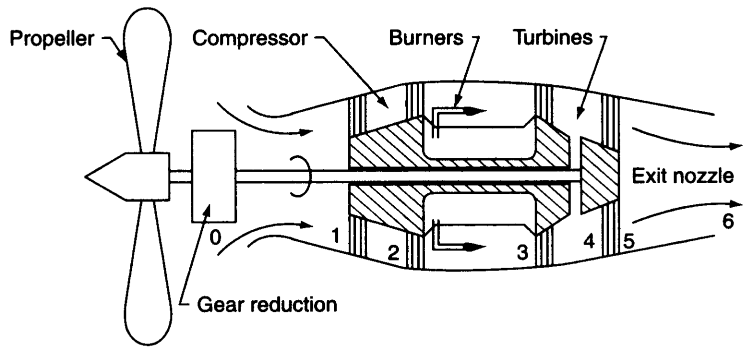
21.15 TURBO-PROP ENGINE

A turbojet engine is quite efficient at flight sppeds above 800 km/h. However, at lower speeds it is not so efficient. This led to the development of turbo-prop engines. A schematic layout of a turbo-prop engine is shown in Fig. 21.31. It comprises of a geared propeller connected to a turbojet engine. The turbine of the turboprop engine is bigger than that of the turbojet engine as it drives both the compressor and the propeller. The propeller consumes about 80 to 90 per cent of the net power available from the turbine and the remaining 10 to 20% of turbine power is left to produce the jet thrust.

Turbo prop engines having two independent turbines have operating convenience for control. One of the turbines drives the compressor, while the other drives the propeller through a reduction gear (Fig. 21.31). The turbine speeds are from 11,000 to 40,000 rpm and propeller speeds are one-tenth to one-twentieth of it. The turboprop engine cycle, Fig. 21.32 is the same as that of the turbojet engine cycle, except that the turbine expansion process is greater. The energy supplied to the propeller, with no losses in the reduction gear, can be written as



Schematic layout of a turbo-prop jet engine



(a)

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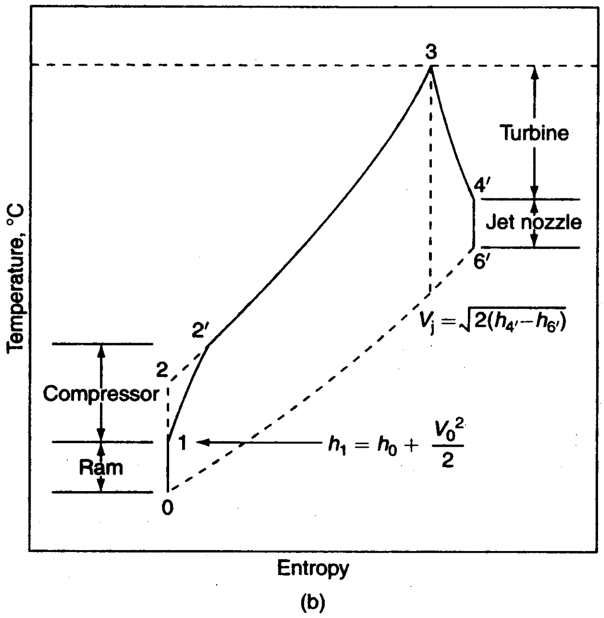
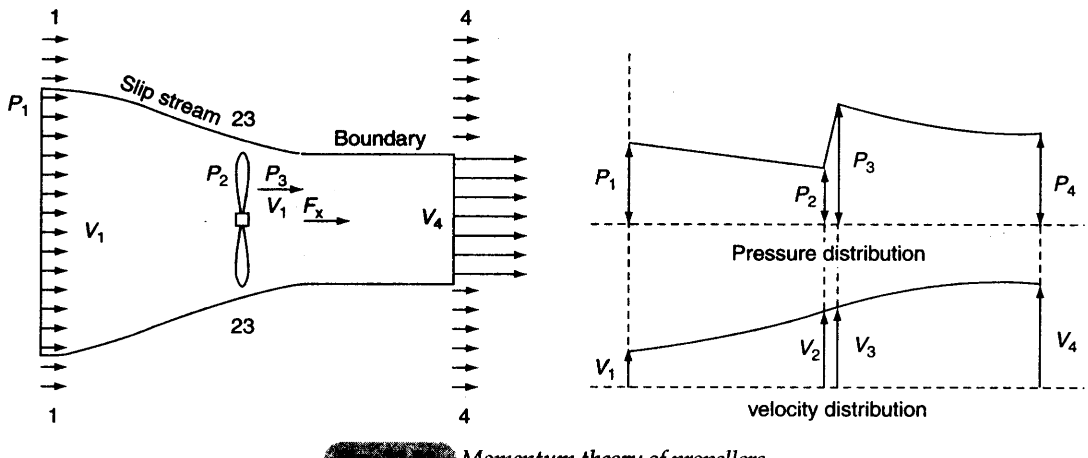


(a) *Two-shaft turbo-prop engine,* (b) *Temperature-entropy diagram for a turboprop engine*

$$\text{Propeller power} = \eta_g \times \eta_p \times \left[(h_3 - h_{4'}) - (h_{2'} - h_1)\right] \frac{\text{kJ}}{\text{kg}} \tag{21.16}$$

where η_g = reduction gear efficiency and η_p= propeller efficiency.

A propeller is used to produce a thrust by changing the momentum of fluid around it. Figure 21.33 shows the flow through a propeller. It is assumed that the flow upstream at the section 1 is undisturbed and the fluid moves towards the propeller with velocity V_1. Since the rotation of blades causes a reduction in pressure on the upstream side of the propeller, the fluid is accelerated towards the propeller and it flows over the



Momentum theory of propellers

propeller with velocity V, and the pressure increases considerably. The velocity at the downstream section 4 is V_4. The body of the fluid affected by the propeller is called 'slipstream', and it the pressure is assumed constant all along the slipstream boundary and at sections 1 and 4. Thus the thrust exerted by the propeller is given by

$$F_x = \rho Q (V_4 - V_1) = (p_3 - p_2) A \tag{21.17}$$

where A = cross-sectional area swept by the propeller $V = \dfrac{V_1 + V_4}{2}$, the average velocity of the upstream and downstream fluids

$$Q = \text{volume flow rate} = A \times V.$$

The useful workdone by the propeller (rate) $P = F \times V = \rho Q (V_4 - V_1) V_1$

Power input to the propeller $= \dfrac{1}{2} \rho Q \left(V_4^2 - V_1^2\right)$

$\therefore$ Propeller efficiency, $\eta_p = \dfrac{\rho Q (V_4 - V_1) V_1}{\frac{1}{2} \rho Q \left(V_4^2 - V_1^2\right)}$

$$= \frac{2 V_1}{V_4 + V_1} = \frac{V_1}{V} \tag{21.18}$$

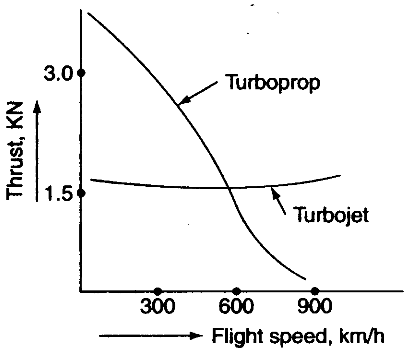


Fig. 21.34 *Variation of thrust with flight speed for a turbojet and a turboprop engine*

Figure 21.34 shows the variation in the thrust produced by a turbo-prop engine with flight velocity. Since the thrust produced at lower flight velocity is considerably higher, the turbo-prop engine has a good take-off characterstics. From Fig. 21.27(a), it is evident that the propulsive efficiency of a turbo-prop engine is higher than that of a turbojet engine at lower flight velocity (up to 800 km/h). The present trend requires higher flight velocities ($M > 1$) and this would require considerable research and experimentation before a turbo-prop will reach its peak performance and service operation.

21.16 BYPASS TURBO-JET ENGINE

A bypass turbojet engine increases the thrust without adversely affecting the propulsive efficiency and fuel economy (Fig. 21.35). There is a fan at the front intake driven by the main shaft. The part of the air drawn by the fan is sent over the combustion chamber (CC) through suitable ducting, to the exhaust unit, thus bypassing the engine. A portion of air is sent to the engine compressor with an added advantage of creating a supercharging effect. The bypass ratio is selected according to the aircraft operational requirement.

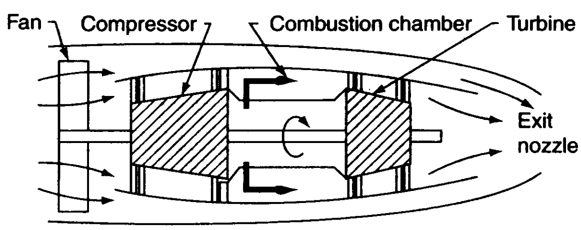


Fig. 21.35 *Bypass turbojet engine*

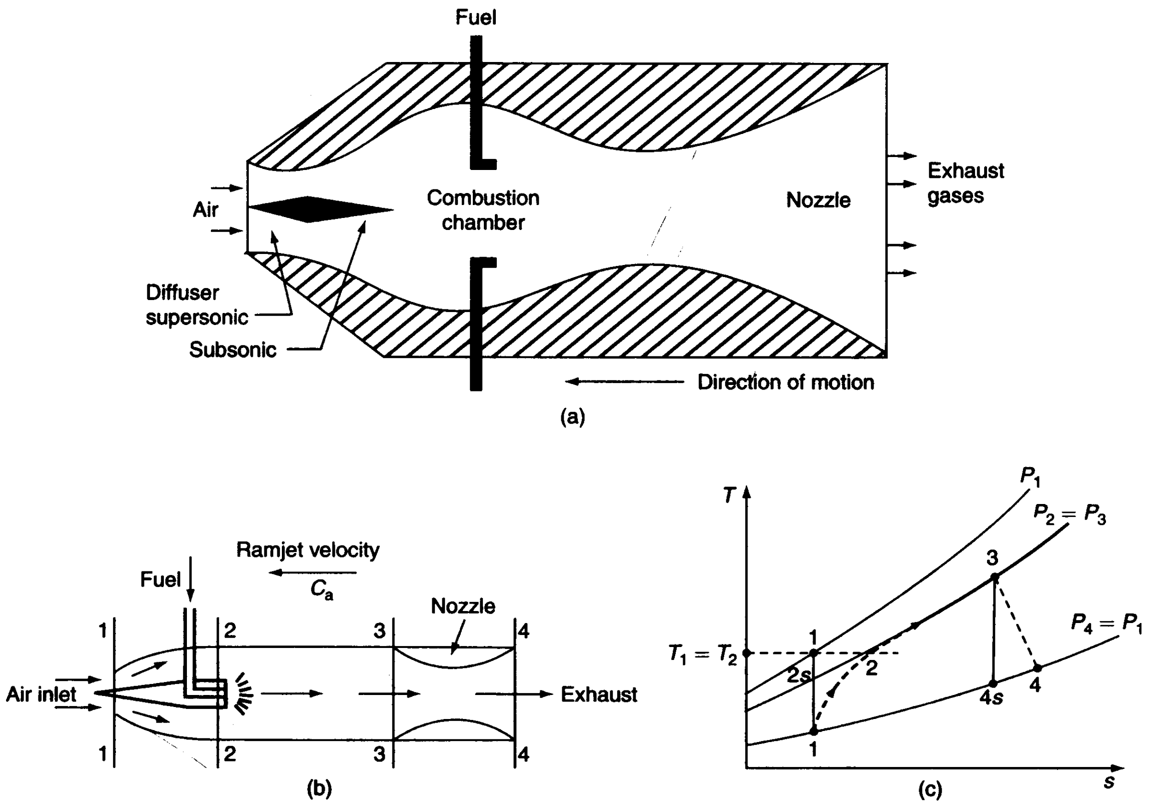
Its advantage is that the lower the velocity of jet efflux, the better the propulsive efficiency and fuel economy, than a straight jet for a given thrust with less K.E. lost to atmosphere. This applies for aircrafts flying at subsonic speeds and for long range. Installation and fire protection are also assisted by the air-cooled duct in the bypass surrounding the hot parts of the engine.

21.17 ATHODYDS-RAMJETS AND PULSEJETS

It is earlier mentioned that at high aircraft speeds, turbojet is designed to take advantage of ram compression. At a Mach number of 2, the ideal ram pressure ratio is nearly 8. At such high ram compression there is no necessity of a mechanical compressor, and so no turbine is required to drive it. Ramjet and pulsejet power plants, called *athodyds* utilize this principle. The word 'athodyd' is derived from the words *aero-thermodynamicduct.*

21.17.1 Ramjet

A ram jet, also known as Lorin tube, is the simplest propulsion device among the air-breathing engine category. It consists of a convergent-divergent diffuser, a combustion chamber and an exit nozzle or tail pipe (Fig. 21.36). In the absence of compressor the compression of air is obtained by the diffuision of the



A Ramjet

high-velocity air stream approaching the diffuser inlet. The velocity is supersonic and the stream is first compressed adiabatically while passing through the normal and oblique shocks. The velocity after the shocks is subsonic while the diffusion takes place in the diverging section of the diffuser, the Mach number being limited to about 0.25 at inlet to the combustion chamber (cc) so that the flame in the CC is stable. The ideal ram pressure ratio increases with the increase in Mach number. (Fig. 21.37) At $M = 2.5$, the pressure ratio p_2/p_1 is about 17 which is very high. However, there will be pressure losses due to shocks, wall friction and flow separation as a result of which this pressure ratio is not obtained.

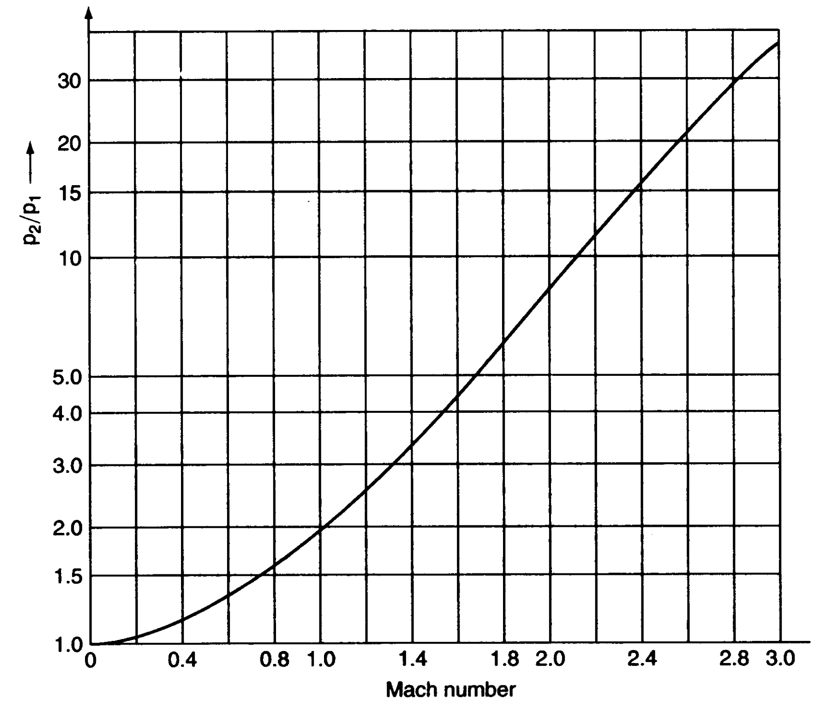
The fuel is sprayed into the CC with injection nozzles and a spark plug initiates the combustion process. The air–fuel ratio is about 15:1 and the temperature after combustion is very high, about 2000 K, much higher than that in the turbojet engines. The products of combustion expand through the nozzle producing the required thrust. In the *T–s* diagram of the ram jet (Fig. 21.38), the combustion process takes place at constant pressure.

For ideal compression and expansion,

$$p_{02}/p_1 = \left(1 + \frac{\gamma - 1}{2} M_i^2\right)^{\frac{\gamma}{\gamma-1}}$$

where p_{02} is the total pressure after isentropic compression, and

$$p_{03}/p_4 = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$



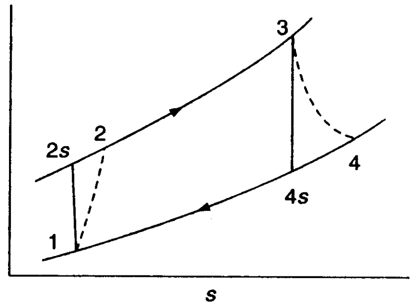
Ideal ram pressure ratio vs. Mach number of vehicles for sea level conditions

Since the compression and expansion ratios are equal,

$$1+\frac{\gamma-1}{2}M_i^2 = 1+\frac{\gamma-1}{2}M_e^2, \quad \text{or} \quad M_i = M_e$$

$$\frac{V_1}{\sqrt{\gamma RT_1}} = \frac{V_e}{\sqrt{\gamma RT_e}}, \quad \text{or} \quad V_e = V_1\left(\frac{T_4}{T_1}\right)^{1/2}$$

Thrust, $$T = \dot{m}_g V_e - \dot{m}_a V_1 \qquad (21.19)$$



T–s plot for Ramjet

21.17.2 Ram-jet Characteristics

1. The system does not contain any moving parts, is light in weight, is less costly and requires no maintenance.
2. The fuel consumption is much lower at higher speeds but is much higher at low and moderate speeds.
3. The system can accept a large variety of fuels.
4. The design of diffuser is quite difficult because the successful operation of ram jet is solely dependent on this convergent-divergent passage.
5. The device is a continuous burning duct with a continuous flow of atmospheric air at high speeds and this requires an elaborate device for fuel injection.
6. The device produces greater thrust per unit weight than any other propulsion device at supersonic speeds except the rockets.
7. The system is not self starting, it cannot accelerate from a rest position and therefore requires a launching device.

21.17.3 Pulse Jet

The pulse jet was developed in Germany during the world war II and is similar to ram jet. It also does not have a compressor and turbine. The basic difference between a ram jet and a pulse jet is that the former is a continuously operating engine working on the Brayton cycle whereas the latter is an intermittent firing engine, the operating cycle may be compared with Otto cycle and is self starting.

The incoming air is compressed in the diffuser. The compressed air flows through mechanically operated non-return valves or shutters. These shutters are opened for incoming air and are closed by the expanding gases coming out of the combustion chamber, Fig. 21.39. The compressed air mixes with fuel and the

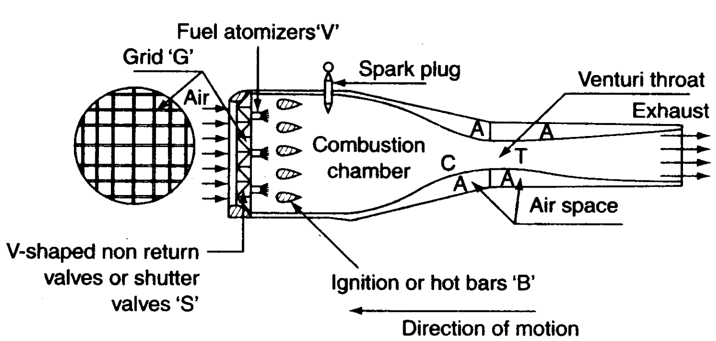


Fig. 21.39 A pulse jet

combustion process is initiated by a spark plug. Once the engine starts operating normally, the spark plug is switched off and the residual flame inside the combustion chamber and hot ignition bars which retain heat from the previous explosion is used for ignition in the succeeding cycles. During combustion the pressure and temperature of the products of combustion increase and when its value is greater than the pressure at the diffuser outlet, the mechanical valves are closed and the gases expand through the nozzle producing the required thrust. After the gases leave the combustion chamber, the pressure decreases and the valves open to allow the compressed air again. The air mixes with fuel and the cycle repeats.

Pulse jet engines are much cheaper than turbo-jet engines and are self starting (compressed air is supplied to the system) but its propulsive efficiency is lower than the turbo-jet engines.

21.18 ROCKET PROPULSION

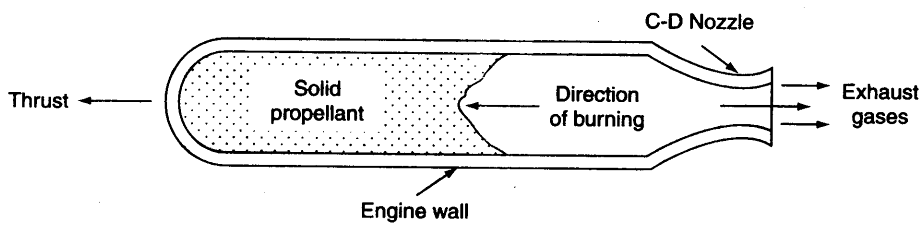
A rocket engine does not draw its oxidiser from the surrounding air. The fuel and the oxidizer, called the *propellant*, is carried by the propelling unit. The function of the rocket is thus independent of speed and has an unlimited operating altitude.

The rocket motors using a chemical fuel are generally divided into two categories: *Solid propellant rocket* and *liquid propellant rocket*. A hybrid rocket combines solid fuel with a liquid oxidizer.

The schematic diagram of a solid propellant rocket is shown in Fig. 21.40. It consists of a seamless tube, usually made of steel, closed at one end. The tube is filled with solid propellant called the '*grain*'. The grain which contains both oxidizer and fuel is electrically ignited and burns fully as there is no method to stop the burning. The open end holds the convergent – divergent nozzle, through which the gases are ejected out at supersonic speed. The reaction to the ejection of the high velocity gases produces the thrust of the rocket motor.

The grain configuration for solid propellant rocket motor varies according to the required thrust-time program for the engine, on which depends the rate of burning. There are two principal types of solid propellants: (a) heterogeneous or composite in which inorganic oxidizer such as potassium perchlorate or sodium nitrate is dispersed in a fuel matrix like organic plastic, asphalt or oil mixtures (75% oxidizer and 25% fuel), and (b) homogeneous propellant consisting of the colloid of nitroglycerine-nitrocellulose: $C_3H_6(NO_3)_3$ – $C_6H_7O_2(NO_3)_3$. The main advantage of solid propellant rockets is its simplicity, having no moving parts and any fuel supply system. The disadvantages are that it has to be large enough to store the entire amount of propellant and is strong enough to withstand high pressures (40 to 140 bar) and temperatures (1600 to 3000°C). There is no provision of cooling. These rockets are suitable for producing thrust for short durations. They are used to power rocket projectiles, guided missiles and as boosters for aircraft as well as spacecraft. The use of additional thrust by rocket motors at take-off is termed jet-assisted take-off (JATO) or rocket-assisted take-off (RATO).

As stated earlier the solid propellant rocket differs from other engines in that total mass of fuel is stored and burned within the combustion chamber. There is no fuel supply system.



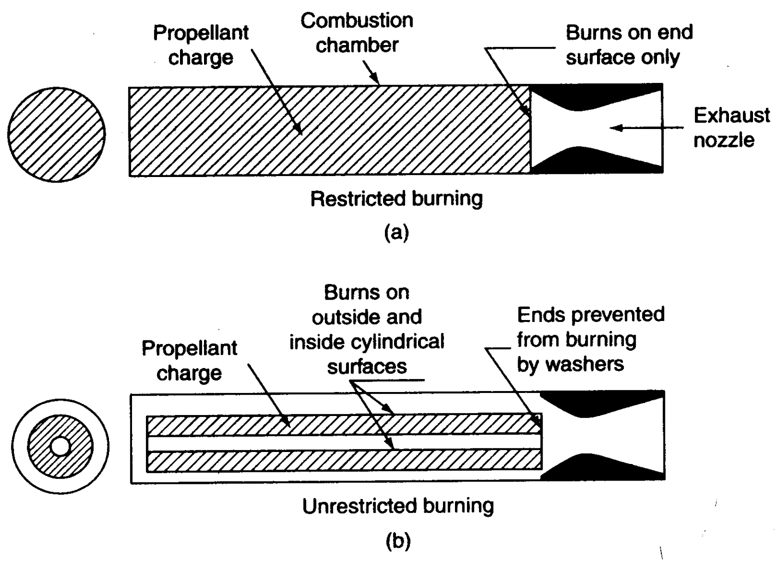
Schematic diagram of a solid propellent rocket

The solid propellant rocket motor consists of a seamless tube, usually made of steel, closed solidly at one end. The open end holds the nozzle which may be a single or multi-orifice type. Usually, small projectile rockets (less than 10 cm in diameter) use single orifice nozzles, while larger ones use multi-orifice nozzles. The single orifice nozzles have their axis parallel to the axis of the motor, while the axis of the orifices of the multi-orifice nozzle may be at an angle to the axis in order to rotate the projectile, thus providing spin stabilization. It should be noted, however, that JATO and large missile booster units are often single nozzle type engines. The solid propellant rockets are divided into two main types according to the amount of surface area exposed to burning. These two types are restricted burning and unrestricted burning.

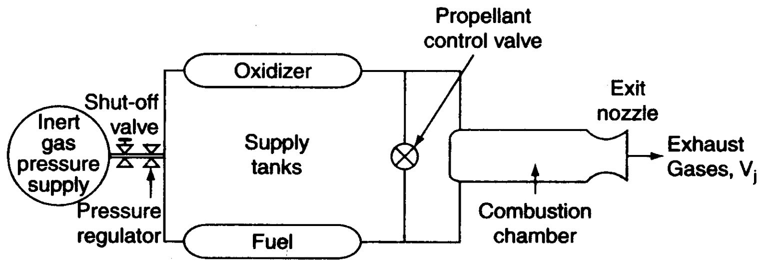
The restricted burning rocket is one in which the propellant is constrained to burn on only one surface, Fig. 21.41(a). The manner in which a cigarette burns is similar to the type of burning of a restricted burning rocket. For example, the charge in a JATO unit is poured in while liquid and fits the chamber tightly so that it is only free to burn on one end. The unrestricted burning rocket is essentially free to burn on all surfaces at the same time, Fig. 21.41(b). The restricted burning rocket delivers a small thrust for a relatively long period while the unrestricted burning rocket delivers a relatively large thrust for a short period. Table 18-1 shows an approximate comparison between the two types of solid propellant rockets.

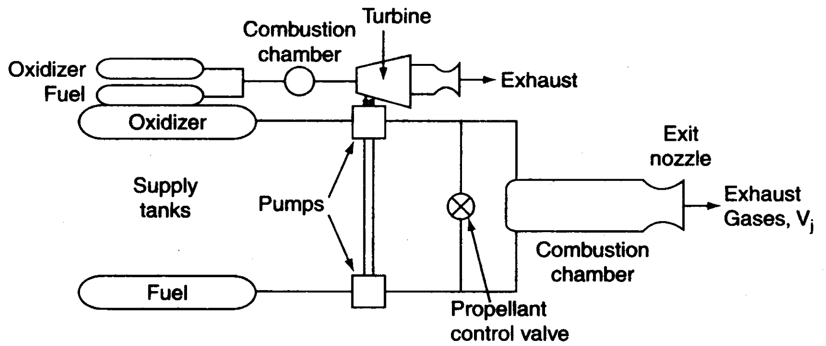
Liquid propellant rockets utilize propellant stores in the container outside the combustion chamber. The liquid propellants are of two types: monopropellant and bipropellant. A monopropellant is a liquid that requires no auxiliary material (oxidizer) for bringing about the release of the thermochemical energy. Nitromethane, propyl nitrate and hydrogen peroxide (H_2O_2) are some monopropellants with only one storage tank. A liquid propellant rocket consists of three major components, viz, (1) rocket motor, (2) propellant system, and (3) controls. An advantage of a liquid propellant is its ability to discontinue combustion at any time.

The *rocket motor* consists of an exit nozzle, Fig. 21.42, a combustion chamber, propellant injectors and an ignition system. As it was stated previously, most liquid fuel rocket motors are cooled by circulating fuel around the engine walls. The cooling fluid may move axially, or may circulate in a helical path around the motor. Normally, *axial flow* is only used for large size motors, because the amount of fuel used for propulsion of small motors is not sufficient to fill the practical size passages that can be manufactured. *Helical flow,* on



Component parts of a solid propellant rocket





Liquid bipropellant rocket systems

the other hand, is usually used in small motors, since it produces too much pressure drop to be practical when considerable quantities of fuel are used as in large size motors.

Film cooling is another method of cooling in which a thin layer of fuel covers the exposed wall surfaces from excessive heat. The thin film of fuel may enter the combustion chamber and nozzle through a series of holes in the motor walls or walls of a special porous material that may be used. This method, however, reduces the efficiency of the engine, because some of the fuel used for cooling is not burned. The best way to arrange for the maintenance of the protective film is not yet known.

The combustion chamber is usually of cylindrical shape (Fig. 21.42) with one end closed and the other end terminating at the entrance to the exit nozzle, which is usually of the DeLaval type. There are no obstructions on the walls between the combustion chamber and the nozzle. In the liquid propellant rocket, the combustion chamber pressures vary from 20 to 50 bar and combustion chamber temperatures from (1650 to 3350)°C. The exhaust gas velocities vary for 1800 to 4000 m/s.

Injection of the fuel and oxidizer into the combustion chamber is accomplished through injectors, which have the same function as those in a compression ignition engine, i.e., atomize and mix the propellants so that a fuel-oxidizer mixture results which can be readily vaporized and burned.

To start a rocket motor an electrical igniter, may be necessary for some propellants. However, propellants which ignite upon contact with oxidizer are more desirable. These self-igniting propellants are called *hypergolic* propellants.

The *propellant system* employs either a pressure feed or a pump feed to transfer fuel from a storage tank to the combustion chamber.

In the *pressure feed* system, Fig. 21.42(a) the pressure exerted by the inert gas stowed under high pressure forces the fuel and the oxidizer through the proportioning valves or orifices that regulate the fuel-oxidizer ratio into the combustion chamber against the combustion pressure, The pressure feed system is simple, inexpensive and reliable. It is limited, however, to small or short duration rockets, because the weight of tanks becomes prohibitive when this system is used in large rockets. The weight of tanks capable of carrying several tons of propellants and being pressurized to pressures of 300 to 750 psia would be greater than the more complex pump feed system described below.

In the *pump feed* system, Fig. 21.42(b), a pump is utilized to force the propellants into the combustion chamber. This feed system is adaptable to rockets of high power and long duration. The pumps are driven by relatively small turbines which may or may not have their own combustion chambers. The turbines having combustion chambers will be operated by the products of combustion of monopropellant (explained later) such as hydrogen peroxide or the main rocket fuel and oxidizer. The turbines having no combustion chambers can be driven by the combustion products bled off from the main rocket motor.

There are advantages and disadvantages to each of these three systems and the choice is based upon factors beyond the scope of this book. It may be of interest, however, to note that the World War II designed V-2 used the monofuel system, and the more recently designed and continuously modified "Viking" rocket uses a chamber bleed system.

A disadvantage in the use of the monopropellant is that the extra weight of the turbine fuel may be from 8 to 5% of the total weight. Since this is a low energy fuel, the over-all efficiency of the rocket is reduced. A disadvantage of the main fuel system is that the rocket fuels themselves are usually too powerful to be used by a turbine, while a disadvantage of the chamber bleed system is that it requires an auxiliary starting system.

21.18.1 Basic Theory of Rockets

Figure 21.43 shows a schematic diagram of the basic components of a liquid bipropellant rocket engine. It consists of an injection system, a combustion chamber, and an exit nozzle. The oxidizer and fuel burn in the CC producing a high pressure. The pressure produced is governed by the weight rate of flow of the propellants, the chemical characteristics of the propellants, and the cross-sectional area of the nozzle throat. The gases are ejected to the atmosphere at supersonic speeds through the nozzle. The nozzle converts the pressure energy of the gases into kinetic energy. The reaction to the ejection of the high-velocity gases produces the thrust of the rocket engine.

The thrust developed is a resultant of the pressure forces acting upon the inner and outer surfaces of the rocket motor. The internal forces acting on the engine are

$$\text{Resultant internal forces} = w_p \cdot V_j + p_j A_j \text{ (newtons)}$$

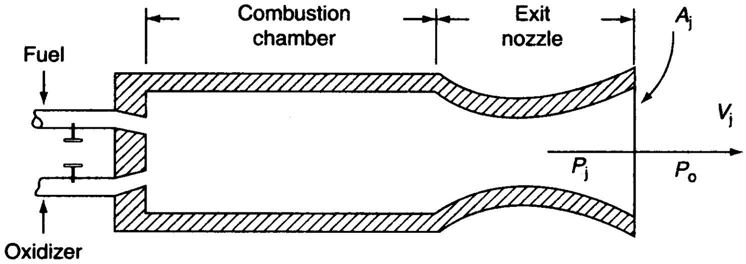


Fig. 21.43 Components of liquid bipropellant rocket motor

where w_p = weight rate of flow, kg/s, V_j = exit jet velocity relative to nozzle, m/s, P_j = exit static pressure, N/m^2 and A_j = exit area, m^2.

The resultant external forces acting on the rocket motor are $p_0 A_j$, where p_0 is the atmospheric pressure, N/m^2. The thrust which is a resultant of the total pressure force, becomes

$$T = w_p V_j + A_j \left(p_j - p_o\right) \text{ newtons} \tag{21.20}$$

The above equation shows the effect of atmospheric pressure on the thrust of a rocket engine. The thrust is maximum when $p_0 = 0$, i.e., operating in vacuum.

In testing a rocket engine, the thrust, the total propellant consumption, and the total time are readily measured. The thrust can be expressed as

$$T = w_p \cdot V_{je} \text{ newtons} \tag{21.21}$$

where V_{je} is the effective jet exit velocity defined by

$$V_{je} = V_j + \frac{A_j (p_j - p_o)}{w_p} \text{ m/s}$$

The thrust power, TP, developed by the rocket engine is

$$\text{TP} = \text{TV}_o = w_p V_{je} V_o \text{ (watts)}$$

The propulsive efficiency, η_p is given by

$$\eta_p = \frac{\text{TP}}{\text{PP}}$$

where the propulsive power is the thrust power plus the kinetic energy lost in the exhaust.

$$\text{K.E. loss} = w_p (V_{je} - V_o)^2 / 2$$

$$\therefore \quad \eta_\phi = \frac{\text{TP}}{\text{TP} + \text{KE loss}} = \frac{w_p V_{je} V_0}{w_p V_{je} V_0 + \left[w_p (V_{je} - V_o)^2 / 2\right]}$$

$$= \frac{2 V_o V_j}{1 + (V_o / V_{je})^2} \tag{21.22}$$

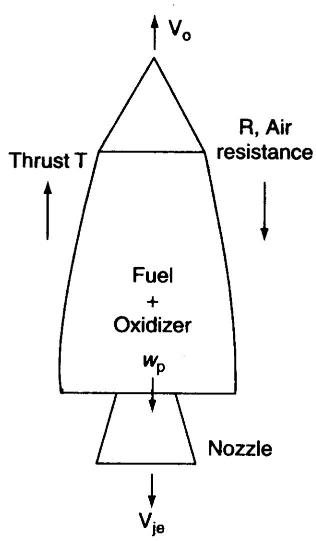


Fig. 21.44 Analysis of rocket acceleration

Specific impulse, I_{sp}, is the thrust produced per unit weight of propellant consumption

$$I_{sp} = \frac{I}{w_p} = V_{je}$$

It is desirable to use propellants with the greatest possible I_{sp}, since this allows a greater useful load to be carried for a given overall rocket weight.

Let us consider an accelerating rocket as shown in Fig. 21.44. It is assumed that R = air resistance, m_1 = initial mass of rocket, w_p = rate at which the propellant burns and m = mass of the rocket and propellant at a given time. Assuming that the pressure at the nozzle exit is the same as the ambient pressure and the momentum of the system does not change with time, the equation of linear momentum can be written as

$$-R - mg + T = ma, \quad \text{where } a = \text{acceleration of the rocket.} \tag{21.23}$$

21.19 NUCLEAR ROCKETS

The main components of a nuclear rocket engine are shown in Fig. 21.45. A nuclear reactor replaces the combustion chamber. Liquid hydrogen is used as working fluid and is heated by the reactor before it expands through the thrust-producing nozzle. A small fraction of the heated gas, propellant, is extracted for driving the turbine which drives the propellant pump.

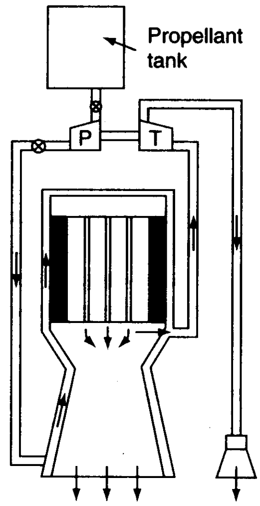


Fig. 21.45 *Main components of a nuclear rocket engine*

Solved Examples

Example 21.1

In a gas turbine plant, air enters the L.P. compressor having a pressure ratio of 3.5 and efficiency 0.85 at 1 bar and 300 K. It then enters the intercooler where it is cooled down to 310 K. The cooled air is further compressed in the H.P. compressor also having a pressure ratio of 3.5 and an efficiency of 0.85. It enters the regenerator having an effectiveness of 0.8. The gases coming out of the combustion chamber enter the H.P. turbine of efficiency 0.88 at 1100 K. The H.P. turbine drives the compressor and there is a reheater between the two turbines. The gases enter the L.P. turbine at 1050 K and the exhaust gases coming out of L.P. turbine of 0.88 efficiency are used to heat the air in the regenerator before leaving to the atmosphere. Determine (a) the power output, and (b) the overall efficiency of the plant. Take c_p for air as 1.005 and for gases as 1.15 kJ/kgK, and γ for air as 1.4 and for gases as 1.33.

Solution The flow and *T–s* diagrams are shown in Fig. Ex. 21.1.

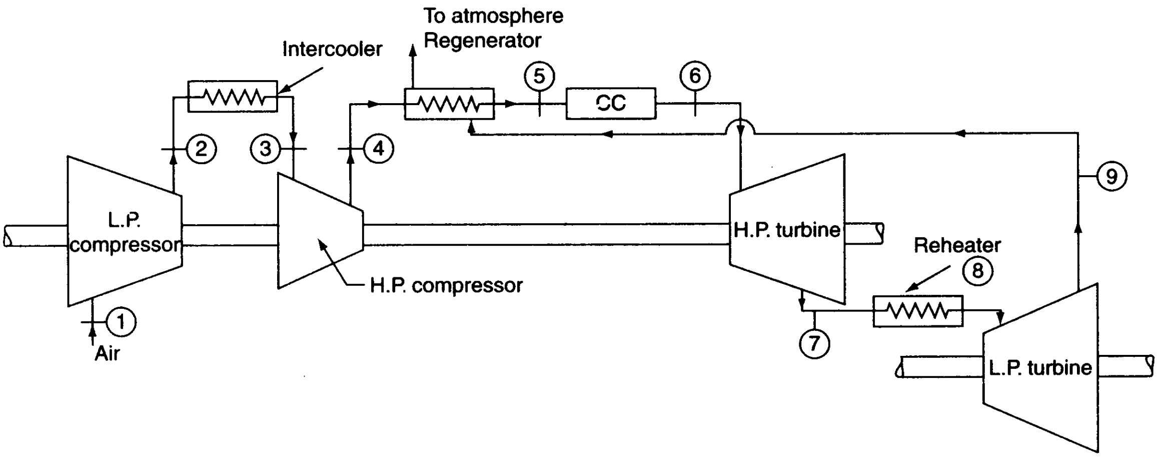
$p_1 = 1$ bar, $T_1 = 300$ K, $p_2 = 3.5$ bar, $T_3 = 310$ K, $p_4 = 12.25$ bar, $T_6 = 1100$ K, $T_8 = 1050$ K

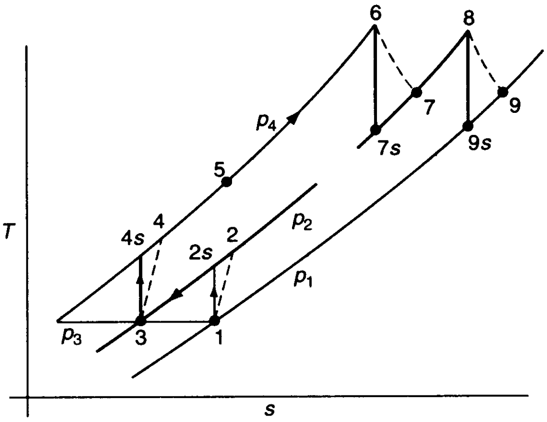
$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad T_{2s} = 300\,(3.5)^{0.286} = 429.26 \text{ K}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 300 + \frac{429.26 - 300}{0.85} = 452.076 \text{ K}$$

$$T_{4s} = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 310(3.5)^{0.286} = 443.5$$





$$T_4 = T_3 + \frac{T_{4s} - T_3}{\eta_c} = 310 + \frac{443.57 - 310}{0.85} = 467.15$$

Power input to the compressor

$$W_c = 1.005\,[(452.076 - 300) + (467.15 - 310)] = 310.76 \text{ kJ/kg}$$

Power output of the HP turbine

$$(W_T)_{HP} = 310.76 = 1.15\,(1100 - T_7)$$

$$\therefore \quad T_7 = 829.77 \text{ K}$$

$$\frac{T_6 - T_7}{T_6 - T_{7s}} = \eta_T = 0.88$$

$$T_{7s} = T_6 - (T_6 - T_7)/\eta_T = 1100 - (1100 - 829.77)/0.88 = 792.92 \text{ K.}$$

Pressure ratio for the H.P. turbine

$$\frac{p_6}{p_7} = \left(\frac{T_6}{T_7}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1100}{792.92}\right)^{\frac{1.33}{0.33}} = 3.74$$

$$\therefore \quad p_7 = \frac{12.25}{3.74} = 3.275 \text{ bar}$$

$$\frac{T_8}{T_{9s}} = \left(\frac{p_7}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (3.275)^{\frac{0.33}{1.33}}$$

$$\therefore \quad T_{9s} = 780.52 \text{ K}$$

$$T_8 - T_9 = (T_8 - T_{9s})\eta_T = (1050 - 780.52) \times 0.88 = 050 - T_9$$

$$\therefore \quad T_9 = 812.86 \text{ K}$$

Power produced by the L.P. turbine

$$(W_T)_{LP} = 1.15\,(1050 - 812.86) = 272.71 \text{ kJ/kg}$$

Total power output = 310.76 + 272.71 = 583.47 kJ/kg

$$\text{Work ratio} = \frac{272.71}{583.47} = 0.467 \text{ (with intercooling)}$$

Q_1 to system having intercooling

$$= (1.15 \times 1100 - 1.005 \times 467.14) + 1.15\,(1050 - 829.77) = 1048.5 \text{ kJ/kg}$$

$$\therefore \quad \eta_{\text{plant}} = \frac{272.71}{1048.5} = 0.26 \quad \text{or} \quad 26\%$$ *Ans.*

Example 21.2

The blade velocity at the mean diameter of a gas turbine stage is 360 m/s. The blade angles at inlet and exit are 20° and 52° respectively and the blades at this section are designed to have a degree of reaction of 50 percent. The mean diameter of the blades is 0.450 m and the mean blade height is 0.08 m. Assuming that the blades are designed according to vortex theory, calculate (a) the flow velocity (b) the blade angles at the tip and the root (c) the degree of reaction at the tip and at the root of the blades.

Solution Given: $\beta_1 = 20° = \alpha_2, \beta_2 = 52° = \alpha_1, V_{b_m} = 360$m/s, $D_m = 0.450$ m, $h_b = 0.08$ m (Fig. Ex. 21.2(a)).

$$V_f \tan\beta_2 - V_f \tan\beta_1 = V_{b_m} = 360$$

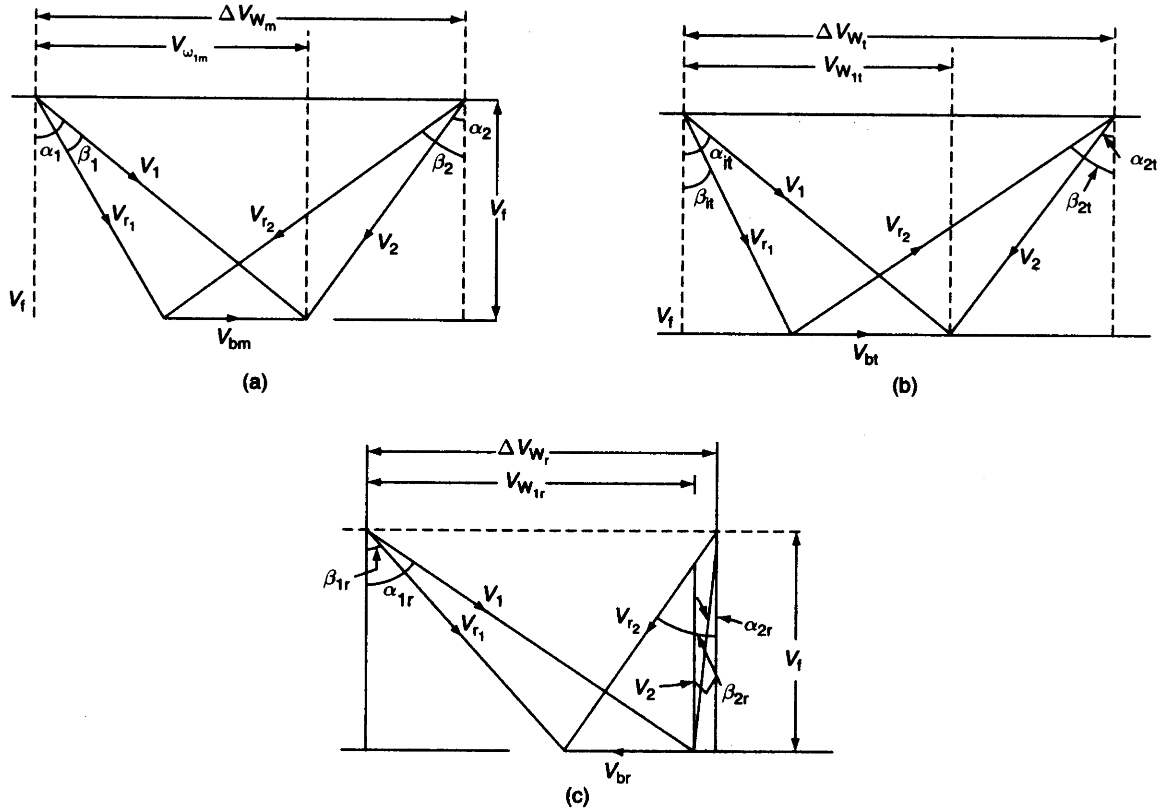
$$V_f(\tan 52° - \tan 20°) = 360$$

$$V_f = \frac{360}{1.2799 - 0.364} = 393 \text{ m/s}$$

The flow velocity at the tip and root is also the same, i.e. 393 m/s. *Ans.* (a)

Blade tip: Using free vortex theory,

$$V_{w_1} r = \text{contant}$$



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$$r_t = \frac{0.45}{2} + 0.04 = 0.265 \text{ m}$$

$$V_{b_t} = \frac{\pi D_t N}{60} = \frac{\pi D_m N}{60} \times \frac{D_t}{D_m}$$

$$= 360 \times \frac{r_t}{r_m} = 360 \times \frac{0.265}{0.225} = 424 \text{ m/s}$$

$$V_{w_{1m}} = V_f \tan\alpha_1 = V_f \tan\beta_2 = 393 \tan 52°$$

$$= 503 \text{ m/s}$$

$$V_{w_{1m}} r_m = V_{w_{1t}} r_t$$

$$V_{w_{1t}} = \frac{503 \times 0.225}{0.265} = 427 \text{ m/s}$$

Using the condition of constant specific work,

$$\Delta V_{w_t} = \frac{\Delta V_{w_m} V_{b_m}}{V_{b_t}} = \frac{V_f(\tan\beta_1 + \tan\beta_2)V_{bm}}{V_{b_t}}$$

$$= \frac{393\left(\tan 20^\circ + \tan 52^\circ\right)\times 360}{424} = 548.57 \text{ m/s}$$

Blade velocity diagram at the blade tip is shown in Fig. Ex. 21.2(b).

$$V_f \tan\alpha_{1_r} = V_{w_{1r}}$$

$$393 \tan\alpha_{1r} = 611.76$$

$$\alpha_{1r} = 57.28^\circ$$

$$V_f \tan\alpha_{2r} = \Delta V_{w_r} - V_{w_{1r}} = 785.79 - 611.76$$

$$= 174.03$$

$$\alpha_{2r} = 23.88^\circ$$

Fixed blades (root): $\alpha_{1r} = 57.28^\circ$, $\alpha_{2r} = 23.88^\circ$ *Ans.* (b)

For moving blades

$$V_f \tan\beta_{1r} = V_{w_{1r}} - V_{b_r} = 611.7 - 296 = 315.76$$

$$\beta_{1r} = 38.78^\circ$$

$$V_f \tan\beta_{2r} = V_{br} + V_f \tan\alpha_{2r} = 296 + 174.03 = 470.03$$

$$\beta_{2r} = 50.1^\circ$$

Moving blades (root): $\beta_{1r} = 38.78^\circ, \beta_{2r} = 50.1^\circ$ *Ans.* (b)

$$V_f \tan\alpha_{1t} = V_{w_{1t}}$$

$$\alpha_{1t} = 47.37^\circ$$

$$V_f \tan\alpha_{2_t} = \Delta V_{w_t} - V_{w_{1t}} = 548.57 - 427 = 121.57$$

$$\tan\alpha_{2t} = \frac{121.57}{393}$$

$$\alpha_{2t} = 17.19^\circ$$

Fixed blades (tip) $\alpha_{1t} = 47.37^\circ, \alpha_{2t} = 17.19^\circ$

Similarly for moving blades,

$$V_f \tan\beta_{1t} = V_{w_{1t}} - V_{b_t}$$

$$393 \tan\beta_{1t} = 427 - 424$$

$$\beta_{1t} = 0.44^\circ$$

$$V_f \tan\beta_{2t} = 424 + 121.57 = 545.57$$

$$\beta_{2t} = 54.23^\circ$$

Moving blades (tip) $\beta_{1t} = 0.44^\circ$, $\beta_{2t} = 54.23^\circ$

Blade root

$$r_r = 0.225 - 0.04 = 0.185 \text{ m}$$

$$V_{b_r} = 360 \times \frac{0.185}{0.225} = 296 \text{ m/s}$$

$$V_{w_{1m}} r_m = V_{w_{1r}} r_r$$

$$V_{w_{1r}} = -\frac{503 \times 0.225}{0.185} = 611.76 \text{ m/s}$$

$$\Delta V_{w_r} = \frac{\Delta V_{w_m} V_{b_m}}{V_{b_r}} = \frac{232593.68}{296} = 7285.79 \text{ m/s}$$

Blade velocity diagram at the blade root is shown in Fig. Ex. 21.2(c)

The degree of reaction is given by

$$R = \frac{V_f(\tan\beta_2 - \tan\beta_1)}{2V_b}$$

At the tip, $R = \dfrac{393(\tan 54.23° - \tan 0.44°)}{2 \times 424} = 0.64$ or 64% *Ans.*

At the root, $R = \dfrac{393(\tan 50.1° - \tan 38.78°)}{2 \times 296} = 0.26$ or 26% *Ans.*

Example 21.3

The products of combustion enter an axial flow gas turbine at 8 bar, 1125 K and leave at 1.5 bar. There are 11 stages, each developing the same specific work with the same stage efficiency. The axial velocity of flow is constant at 110 m/s and the polytropic efficiency is 0.85. At a particular stage, the mean blade velocity is 140 m/s, the stage has 50% reaction at the mean blade height and the specific work output is constant across the stage at all radii. Assuming that $\gamma = 1.33$ and $c_p = 1.15$ kJ/kgK for gases, estimate (a) the blade angles at inlet and exit, (b) the overall isentropic efficiency of the turbine, and (c) the stage efficiency.

Solution Polytropic efficiency η_p is defined by

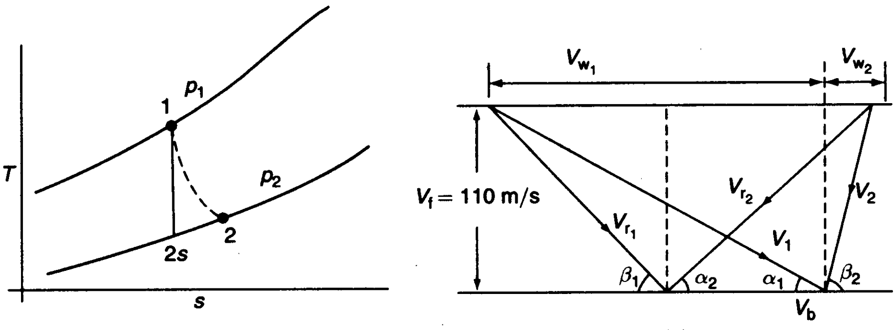
$$\eta_p = \frac{dh}{dh_s} = \frac{c_p dT}{vp} \cong \frac{c_p dT}{\frac{RT}{p} dp}$$

$$\eta_p \int_1^2 \frac{dp}{p} = \frac{c_p}{R} \int_1^2 \frac{dT}{T}$$

$$\eta_p \ln \frac{p_2}{p_1} = \frac{\gamma}{\gamma - 1} \ln \frac{T_2}{T_1}$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_p}{\gamma}}$$

$$T_2 = 1125\left(\frac{1.5}{8}\right)^{0.2125} = 788.25 \text{ K}$$



(a) (b)

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For isentropic expansion,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.5}{8}\right)^{0.25} = 0.658$$

$\therefore$ $$T_{2s} = 1125 \times 0.658 = 740.29 \text{ K}$$

$\therefore$ overall isentropic efficiency of the turbine

$$\eta_s = \frac{T_1 - T_2}{T_1 - T_{2s}} = \frac{1125 - 788.25}{1125 - 740.29} = 0.875 \quad \text{or} \quad 87.5\%$$ *Ans.* (b)

Work output of the turbine

$$W_T = c_p(T_1 - T_2) = 1.15\,(1125 - 788.25) = 387.26 \text{ kJ/kg}$$

$\therefore$ work output per stage $\frac{387.26}{11} = 35.2$ kJ/kg

From the velocity triangles at inlet and exit of the blades,

$$V_1 = V_{r_2}, V_{r_1} = V_2$$

Work output per stage $= \left(V_{w_1} + V_{w_2}\right)V_b = 35.2 \times 10^3$ J/kg

$\therefore$ $$V_{w_1} + V_{w_2} = \frac{35.2 \times 10^3}{140} = 251.43$$

$$V_{w_2} = V_{w_1} - V_b$$

$$V_{w_1} + V_{w_1} - V_b = 251.43$$

$\therefore$ $$V_{w_1} = \frac{251.43 + 140}{2} = 195.71 \text{ m/s}$$

$$\alpha_1 = \tan^{-1}\frac{110}{195.71} = 29.33° = \alpha_2$$

These are the blade outlet angles measured with the direction of rotation.

$$\beta_1 = \tan^{-1}\frac{110}{195.71-140} = 63.14° \text{ (blade angle from direction of rotation)}$$ *Ans.* (a)

Enthalpy drop per stage = 35.20 kJ/kg

$\therefore$ Temperature drop per stage $= \dfrac{35.2}{1.15} = 30.7$ K

$\therefore$ $T_1' = 1125 - 30.7 = 1094.3 \text{ K} = T_1 - T_1'$

Now $$\frac{T_1'}{T_1} = \left(\frac{p_1'}{p_1}\right)^{\frac{(\gamma-1)\eta_p}{\gamma}} = \left(\frac{p_1'}{p_1}\right)^{0.2125}$$

$\therefore$ $$\frac{p_1'}{p_1} = \left(\frac{1094.3}{1125}\right)^{\frac{1}{0.2125}} = 0.878$$

Isentropic temperature ratio

$$\frac{T_{1s}}{T_1} = (0.878)^{\frac{\gamma-1}{\gamma}} = 0.968$$

$$T_{1s} = 1125 \times 0.968 = 1089.1 \text{ K}$$

$\therefore$ Stage efficiency, $\eta_{st} = \dfrac{1125-1094.3}{1125-1089.1} = 0.853$ or 85.3% *Ans.* (c)

Example 21.4

A turbojet aircraft is flying at 800 km/h at 10,700 m where the pressure and temperature of the atmosphere are 0.24 bar and − 50°C respectively. The compressor pressure ratio is 10/1 and the maximum cycle temperature is 1093 K. Calculate the thrust developed and the specific fuel consumption using the following particulars: entry duct efficiency = 0.9, isentropic efficiency of the compressor = 0.9, stagnation pressure loss in the combustion chamber = 0.14 bar, calorific value of fuel = 43.3 MJ/kg, combustion efficiency = 0.98, isentropic efficiency of turbine = 0.92, mechanical efficiency of drive = 0.98, jet pipe efficiency = 0.92, nozzle outlet area = 0.08 m², c_p = 1.005 KJ/kg K and γ = 1.4 for air, c_p = 1.15 kJ/kg K and γ = 1.333. Assume that the nozzle is convergent.

Solution K.E. of air at inlet $= \dfrac{1}{2} \times \left(\dfrac{800 \times 1000}{3600}\right)^2 = \dfrac{1}{2} \times (222.2)^2$ Nm/kg = 24.7 kJ/kg

$$T_{01} - T_0 = \frac{24.7}{1.005} = 24.6 \text{ K}$$

$\therefore$ $T_{01} = [(-50) + 273] + 24.6 = 247.6$ K

Intake efficiency, $0.9 = \dfrac{T_{01s} - T_0}{T_{01} - T_0}$

$$T_{01s} - T_0 = 0.9 \times 24.6 = 22.1 \text{ K}$$

$\therefore$ $$T_{01s} = 22.1 + (-50 + 273) = 245.1 \text{ K}$$

$$\frac{p_{01}}{p_0} = \left(\frac{T_{01s}}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{245.1}{223}\right)^{\frac{1.4}{0.4}} = (1.1)^{3.5} = 1.393$$

$\therefore$ $$p_{01} = 1.393 \times 0.24 = 0.334 \text{ bar}$$

For the compressor,

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma}{\gamma-1}} = (10)^{\frac{0.4}{1.4}} = 1.931$$

$\therefore$ $$T_{02s} = 247.6 \times 1.931 = 478 \text{ K}$$

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = 0.9$$

$$T_{02} - T_{01} = \frac{478 - 247.6}{0.9} = 256 \text{ K}$$

$$T_{02} = 247.6 + 256 = 503.6 \text{ K}$$

$$p_{02} = 10 \times p_{01} = 10 \times 0.334 = 3.34 \text{ bar}$$

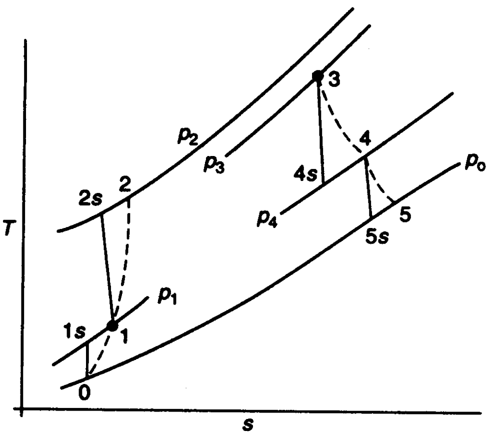
$$p_{03} = 3.34 - 0.14 = 3.2 \text{ bar}$$

Since $W_c = W_T$, by energy balance

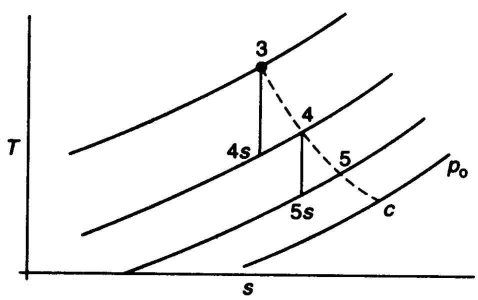
$$T_{03} - T_{04} = \frac{1.005\,(503.6 - 247.6)}{1.15 \times 0.98} = 228.3 \text{ K}$$

$$T_{04} = 1093 - 228.3 = 864.7 \text{ K}$$

$$\eta_T = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}} = 0.92$$



(a)



(b)

$$T_{03} - T_{04s} = \frac{228.3}{0.92} = 248.2 \text{ K}$$

$$T_{04s} = 1093 - 248.2 = 844.8 \text{ K}$$

$$\frac{p_{03}}{p_{04}} = \left(\frac{T_{03}}{T_{04s}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1093}{844.8}\right)^{\frac{1.333}{0.333}} = 2.803$$

$$\therefore \qquad p_{04} = \frac{3.2}{2.803} = 1.156 \text{ bar}$$

For choked flow in the nozzle, the critical pressure ratio is given by

$$\frac{p_c}{p_{04}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.333}\right)^{\frac{1.333}{0.333}} = 0.54$$

$$p_{cr} = 0.54 \times 1.156 = 0.624 \text{ bar}$$

Since the atmospheric pressure is 0.24 bar, the nozzle is choked and hence the nozzle exit velocity is sonic (Fig. Ex. 21.4(b)).

$$\frac{T_5}{T_{04}} = \frac{2}{\gamma+1} = \frac{2}{2.333} = \text{T}_5 = 864.77 \times 0.857 = 741.33 \text{ K}$$

$$\text{Jet pipe efficiency} = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}} = 0.92$$

$$T_{05s} = 864.7 - \frac{864.7 - 741.33}{0.92} = 730.6 \text{ K}$$

$$\frac{p_{05}}{p_5} = \left(\frac{T_{05}}{T_{05s}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{864.7}{730.6}\right)^{\frac{1.333}{0.333}} = 1.963$$

$$p_5 = \frac{1.156}{1.963} = 0.589 \text{ bar}$$

$$R = \frac{c_p(\gamma-1)}{\gamma} = \frac{1.15 \times 0.333}{1.333} = 0.2873 \text{ kJ/kg K}$$

$$v_5 = \frac{RT_5}{p_5} = \frac{0.2873 \times 741.3}{0.589 \times 10^5} = 3.616 \text{ m}^3\text{/kg}$$

Jet velocity, $V_j = \sqrt{\gamma RT_5} = \sqrt{1.333 \times 287.3 \times 741.3} = 532.8$ m/s

Mass flow rate, $\dot{m} = \dfrac{0.08 \times 532.8}{3.616} = 11.79$ kg/s

Momentum thrust $= \dot{m}\left(V_j - V_o\right) = 11.79\,(532.8 - 222.2) = 3661$ N

Pressure thrust $= (p_5 - p_0)\,A = (0.589 - 0.24) \times 0.08 \times 10^5 = 2792$ N

$\therefore$ total thrust $= 3661 + 2792 = 6453$ N *Ans.*

Heat supplied, $Q_1 = \dot{m}c_{p_g}(T_{03} - T_{02})$

$$= 11.79 \times 1.15\,(1093 - 503.6) = 7990 \text{ kW}$$

$$Q_1 = \dot{m}_f \times C.V. \times \eta_{cc}$$

$\therefore$ Fuel burning rate, $\dot{m}_f = \dfrac{7990}{43{,}300 \times 0.98} = 0.188$ kg/s

$\therefore$ Specific fuel consumption $= \dfrac{0.188 \times 10^3}{6453} = 0.0291 \dfrac{\text{kg}}{\text{KNs}}$ *Ans.*

Because of the low value of fuel – air ratio, one can assume that the mass flow rate of air is equal to the mass flow rate of gases.

Example 21.5

A turbojet flying at 850 km/h has an air mass flow rate of 50 kg/s. The enthalpy drop across the nozzle is 200 kJ/kg and the nozzle efficiency is 0.9. The air fuel ratio is 80 and the heating value of the fuel is 40 MJ/kg. Estimate the propulsive power, thrust power, propulsive and thermal and overall efficiency of the unit.

Solution Velocity of aircraft, $V_0 = \dfrac{850 \times 1000}{3600} = 236.11$ m/s

A/F ratio $= 80.$

$\therefore$ $\dot{m}_f = \dfrac{50}{80} = 0.625$ kg/s

Velocity of gases at exit from the nozzle, V_e.

$$= \sqrt{2c_p(\Delta h)\eta_n} = \sqrt{2 \times 1005 \times 200 \times 0.9} = 601.5 \text{ m/s}$$

Thrust, $T = \dot{m}_g V_e - \dot{m}_a V_0 = 50.625 \times 601.5 - 50 \times 236.11 = 18.645$ kN

$\therefore$ Thrust power (TP) = TV_0 = 18.645 × 236.11 = 4402.37 kW *Ans.*

Propulsive power (pp) is the energy required to change the momentum of the fluid and is expressed by the difference in K.E. as given by

$$\text{PP} = \frac{1}{2}(\dot{m}_a + \dot{m}_f)V_e^2 - \frac{1}{2}m_a V_o^2$$

$$= \frac{1}{2}(50.625) \times (601.5)^2 - \frac{1}{2} \times 50 \times (236.11)^2$$

$$= 7.72 \text{ MW}$$ *Ans.*

Propulsive efficiency, $\eta_p = \dfrac{\text{TP}}{\text{PP}} = \dfrac{4.402}{7.72} = 0.567$ or 56.7 *Ans.*

Thermal efficiency $= \dfrac{\text{PP}}{\dot{m}_f \times \text{CV}} = \dfrac{7.72}{0.625 \times 40} = 0.31$ or 31% *Ans.*

Overall efficiency $= \eta_{th} \times \eta_p = 0.31 \times 0.567 = 0.176$ or 17.6% *Ans.*

Example 21.6

Air at the ambient conditions of 0.56 bar and 260 K enters the compressor having a pressure ratio of 6 and an efficiency of 0.85 of a turboprop aircraft flying at a speed of 360 km/h. The propeller diameter is 3 m, propeller efficiency 0.8 and gear reduction efficiency is 0.95. The products of combustion enter the gas turbine having an expansion ratio of 5 and an efficiency of 0.88 at 1100 K. Determine the air – fuel ratio, thrust produced by the nozzle with an efficiency of 0.9, thrust by the propeller and the mass flow rate of air flowing through the compressor. Given: C. V. of fuel = 40 MJ/kg.

Solution Velocity of aircraft, $V_0 = \dfrac{360 \times 10^3}{3600} = 100$ m/s

Compressor: $p_1 = 0.56$ bar, $p_2 = 6 \times 0.56 = 3.36$ bar,

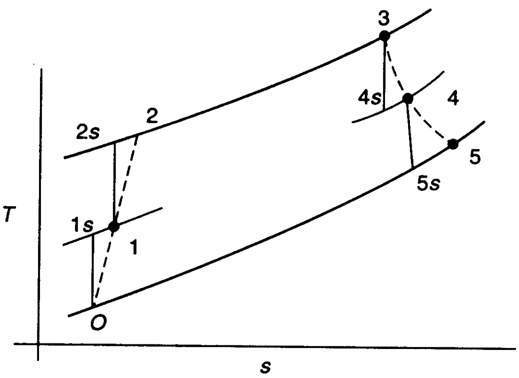
$$\frac{T_{2s}}{T_1} = 6^{0.286} = 1.716, \quad T_{2s} = 260 \times 1.716 = 446.16 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = 0.85, \quad T_2 = 260 + \frac{446.16 - 260}{0.85} = 479.01 \text{ K}$$

Power input to compressor

$W_c = c_p (T_2 - T_1)$

$= 1.003\,(479.01 - 260) = 220.1$ kJ/kg



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Combustion chamber

By making an energy balance,

$$C_{p_a} T_2 + \dot{m}_f \times \text{C.V.} = C_{p_g} T_3 (1 + m_f)$$

Assuming $C_{p_a} = C_{p_g}$,

$$479.01 + \dot{m}_f \times \frac{40{,}000}{1.005} = 1100(1 + \dot{m}_f)$$

$\therefore$ $\dot{m}_f = 0.016$ kg/s

$\therefore$ $\text{air} - \text{fuel ratio} = \dfrac{1}{\dot{m}_f} = \dfrac{1}{0.016} = 62.23$ *Ans.*

Turbine

$$T_3 = 1100 \text{ K}, p_3 = 6 \times 0.56 = 3.36 \text{ bar}$$

$$p_4 = \frac{3.36}{5} = 0.672 \text{ bar}$$

Temperature at the end of isentropic expansion

$$T_{4s} = T_3 / 5^{0.286} = 644.44 \text{ K}$$

$$T_4 = T_3 - (T_3 - T_{4s})/\eta_T = 1100 - (1100 - 644.44)/0.88$$
$$= 699.11 \text{ K}$$

Power output of turbine, $W_T = (1 + m_f)C_p(T_3 - T_4)$

$= 1.016 \times 1.005\,(1100 - 699.11)$

$= 409.34$ kJ/kg air

Power available for propeller $= 409.34 - 220.1 = 189.24$ kJ/kg

Nozzle

$$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{0.286} = \left(\frac{0.672}{0.56}\right)^{0.286} = 1.053$$

$$T_{5s} = \frac{699.11}{1.053} = 613.36 \text{ K}$$

Velocity of gases at nozzle exit

$$V_j = \sqrt{2 \times 1.005 \times 0.9(699.11 - 663.36)} = 254.31 \text{ m/s}$$

Thrust by jet $= m_g V_j - m_a V_\phi = 1.016 \times 254.31 - 1 \times 100$

$= 158.38$ N/kg air

Propeller

Propeller efficiency $= 0.8 = \dfrac{V_0}{V}$

$$\therefore \quad V = \frac{100}{0.8} = 125 \text{ m/s}$$

$$V = \frac{V_4 + V_0}{2} = 125$$

$$\therefore \quad V_4 = 250 - 100 = 150 \text{ m/s}$$

$$Q = \frac{\pi}{4}d^2 \times V = \frac{\pi}{4}(3)^2 \times 125 = 883.7 \text{ m}^3/\text{s}$$

Thrust power of the propeller

$$= \frac{1}{2}PQ\left(V_4^2 - V_0^2\right) = \frac{1}{2} \times \frac{0.56 \times 10^5}{287 \times 260} \times 883.7\left(150^2 - 100^2\right)$$

$$= 4142.3\,\frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{\text{s}} \times \frac{\text{m}^2}{\text{s}^2} = 4142.3 \text{ kJ/s}$$ *Ans.*

Reduction gear efficiency $= 0.95$

Power to be supplied by the turbine $= \dfrac{4142.3}{0.95} = 4360.3$ kW

Mass flow rate through the compressor

$$\dot{m}_a = \frac{4360.3 \text{ kW}}{189.23 \text{ kJ/kg}} = 23.04 \text{ kg/s}$$ *Ans.*

Thrust produced by the propeller $= \dfrac{4142.3 \times 0.8}{100} = 33.14$ kN *Ans.*

Example 21.7

A rocket is fixed vertically starting from rest. The initial mass of the rocket is 15000 kg and the stored propellant burns at the rate of 125 kg/s. The gases come out at a velocity of 2000 m/s relative to the rocket. Neglecting air resistance, calculate (a) the velocity it will attain in 70 seconds, and (b) the maximum height that the rocket will attain.

Solution Neglecting air resistance, the linear momentum equation (Eq. 21.23) can be written as

$$T - mg = m\,\mathrm{d}V/\mathrm{d}t \text{ and } m = m_i - \dot{m}_p t$$

where m_i = initial mass of rocket and $\dot{m}_p$ = rate of burning fuel.

$$\mathrm{d}V = \frac{T - (m_i - \dot{m}_p t)g}{m_i - \dot{m}_p t}\,\mathrm{d}t$$

$$= \frac{T}{m_i - \dot{m}_p t}\,\mathrm{d}t - g\,\mathrm{d}t$$

On integration,

$$V = -\frac{T}{\dot{m}_p}\ln\left(1 - \frac{\dot{m}_p t}{m_i}\right) - gt$$

since $T = \dot{m}_p V_e$, $\quad V_e = T/\dot{m}_p = 2000$ m/s

After 70 seconds, the velocity of the rocket will be

$$V = -2000\ln\left(1 - \frac{125 \times 70}{15000}\right) - 9.81 \times 70$$

$$= 1064.24 \text{ m/s}$$ *Ans.* (a)

The height travelled during the first 70 seconds,

$$h_1 = \int_0^t v\,\mathrm{d}t = \int_0^{70}\left[-2000\ln\left(1 - \frac{125t}{15000}\right) - gt\right]\mathrm{d}t$$

$$= 28.4 \text{ km}$$

The rocket will be assumed to continue its flight upward until its K.E. is converted to P.E.

$$\frac{1}{2}mV^2 = mgh_2$$

$$\therefore \quad h_2 = (1064.24)^2/2 \times 9.81$$

$$= 57.73 \text{ km.}$$

$$\therefore \quad \text{Maximum height} = 28.4 + 57.73$$

$$= 86.13 \text{ km.}$$ *Ans.*

Example 21.8

The pressure and temperature developed in the combustion chamber of a chemical rocket engine are 2.4 MPa and 3170 K respectively. The atmosperic pressure is 55 kPa. The pressure at the nozzle exit is 85 kPa and the cross-sectional area at the nozzle throat is 0.06 m^2. The nozzle efficiency is 0.91, the coefficient of discharge is 0.98, $\gamma = 1.25$, $R = 0.693$ kJ/kgk, the half angle of divergence is 12: Determine the thrust and the specific impulse.

Solution Jet velocity, $V_j = \sqrt{2(h_c - h_j)} = \sqrt{2c_p(T_c - T_j)}$

$$= \sqrt{2c_pT_c\left[1-\left(\frac{p_j}{p_\gamma}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{p_j}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2\times1.25\times0.693\times3170}{0.25}\left[1-\left(\frac{55}{2400}\right)^{\frac{0.25}{1.25}}\right]}$$

$= 3271.94$ m/s.

The thrust developed by the nozzle is directly proportional to the axial velocity, a correction factor is used to account for the nonaxial flow of gases. The correction factor is given by $k = (1+\cos\alpha)/2$, where α is the semidivergence angle. The actual velocity of gases is

$$(V_j)_{act} = \sqrt{\eta_n}\,.\,k \times \text{theoretical velocity}$$

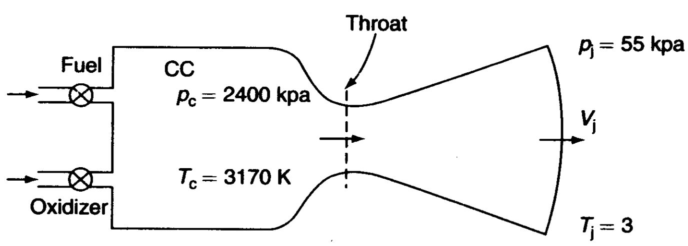
$$\therefore \quad (V_j)_{act} = \sqrt{0.91}\times(1+\cos 12^\circ)/2 \times 3271.94$$

$= 3086.9$ m/s.

The velocity at the throat is sonic.

The mass flow rate, $\dot{m}$

$$= P_tA_tV_t = A_tP_c\left[\frac{\gamma}{RT_c}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$$



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(see chapter 17)

$$= 0.06 \times 2.4 \times 10^6 \left[\frac{1.25}{693 \times 3170} \left(\frac{2}{2.25} \right)^{\frac{2.25}{0.25}} \right]^{\frac{1}{2}} = 63.93 \text{ kg/s}$$

Actual mass flow rate = 0.98 × 63.93 = 62.66 kg/s

$$(\dot{m})_{act} = P_e A_e V_j$$

which gives $A_e = 0.2589 \text{ m}^2$

Thrust produced $= \dot{m} V_j + A_e (p_e - p_o)$

$= 62.66 \times 3086.9 + 0.2589\,(85 - 55) \times 10^3$

$= 201.18$ kN *Ans.*

$$\text{Specific impulse} = \frac{201.18 \times 10^3}{62.66} = 3210 \text{N/kg/s}$$ *Ans.*

Review Questions

21.1 What are the applications of gas turbines?

21.2 How does the open-cycle GT plant compare with the closed-cycle GT plant?

21.3 Explain the advantages of a GT plant.

21.4 Give an analysis of the GT plant. What is the effect of pressure ratio on GT plant output and efficiency?

21.5 Derive the expression of optimum pressure ratio for a GT plant having compressor efficiency η_c and turbine efficiency η_T.

21.6 What is work ratio of a GT plant ? Show that it is

$$r_w = 1 - \frac{T_1}{T_3} r_p^{\frac{\gamma-1}{\gamma}}$$

21.7 Explain the effect of regeneration on the performance of a GT plant with sketches. What is its effectiveness?

21.8 Explain the effects of intercooling and reheating in a GT plant.

21.9 Draw the schematic diagram of a GT plant with intercooling, reheating and regeneration and define the overall plant efficiency.

21.10 What is a semi-closed cycle GT plant?

21.11 Discuss the GT plant performance under variable loads.

21.12 Compare centrifugal compressor with an axial flow compressor.

21.13 Explain with suitable sketches the operation of combustion chambers in a GT plant.

21.14 Define combustion efficiency. What are the performance criteria of a CC?

21.15 Explain the cooling of GT blades. What is degree of reaction ? Draw the velocity triangles at inlet and exit of the blades.

21.16 Explain vortex blading. What are the basic requirements for a gas turbine?

21.17 Explain the effect of water injection on gas turbine performance.

21.18 Discuss the use of different fuels in a GT plant.

21.19 Discuss on the different materials used for GT plant construction.

21.20 On which laws does the jet propulsion system depend? Explain.

21.21 What is the propulsive force? What is thrust?

21.22 Enlist the various propulsive devices for aircrafts and missiles.

21.23 How does a turboprop differ from a turbojet engine? Explain propulsive power and propulsive efficiency.

21.24 Explain the operation of a propeller. What is an athodyd ? What is a rocket ?

21.25 Explain with a neat sketch the operation of a turbojet engine. Draw the T–s diagram.

21.26 Define propulsive power, thrust power and propulsive efficiency.

21.27 Explain the effect of flight speed on propulsive efficiency and the thrust power.

21.28 What is the effect of afterburner in a turbojet engine? Explain with a neat sketch.

21.29 Explain the operation of a turboprop engine with neat sketches.

21.30 Explain the momentum theory of propellers. What is the effect of flight speed on the thrust for a turbojet and a turboprop engine?

21.31 What is a bypass turbojet engine? What is its advantage?

21.32 Explain the operation of a ramjet engine. What is the thrust produced?

21.33 Explain with a sketch the operation of a pulse-jet engine.

21.34 What is the principle of operation of rocket propulsion? What is 'grain'?

21.35 What do you mean by JATO and RATO?

21.36 What are restricted and unrestricled burning of a solid propellant rocket?

21.37 Explain the operation of liquid propellant rocket with sketches. What is a hypergolic propellant ?

21.38 Derive the expression for thrust produced in a rocket. What is propulsive efficiency? What is specific impulse?

21.39 Explain a nuclear rocket engine with a sketch.

21.40 What do you mean by je thrust and pressure thrust in aircrafts and missiles ? Derive the expression for total thrust in a subsonic aircraft.

Problems

21.1 Estimate the pressure ratio at which the specific output is maximum if the temperature at the compressor inlet is 300 K and that at the turbine inlet is 1000 K. The isentropic efficiencies of the compressor and turbine are 0.85 and 0.88 relatively. Also, obtain the mass flow rate per kWh of air and fuel if the calorific value of fuel is 40 MJ/kg.

$\left(Ans.\ \gamma_p = 5, \dot{m}_a = 30.27 \text{ kg/kwh}, \dot{m}_f = 0.375 \text{ kg/kwh}\right)$

21.2 The pressure ratio of the compressor in a GT plant is 7. Air enters the compressor at 1 bar, 25°C, and the temperature at the turbine inlet is 1050 K. The gases leave the turbine at 1 bar. The combustion efficiency is 0.89 and the pressure drop in the combustion chamber is 0.11 bar. Taking $\eta_c =$ 0.82 and $\eta_T = 0.85$, estimate (a) the thermal efficiency of the plant, (b) the work ratio, and (c) the specific fuel consumption. The heating value of the fuel is 42 MJ/kg. Take C_p for air as 1.005 and C_p for gases as 1.11 kJ/kgK.

(*Ans.* (a) 22.53%, (b) 0.3633, (c) 0.38 kg/kwh)

21.3 Air at 1 bar, 300 K enters the compressor of a gas turbine unit and comes out at 8 bar. The compressor is driven by an H.P. turbine, and the L.P. turbine drive a generator. The isentropic efficiencies of the compressor, H.P. turbine and L.P. turbine are 0.82, 0.85 and 0.85 respectively. Determine the net power developed, the work ratio and the overall efficiency of the unit if the air flow rate is 5 kg/s, the air – fuel ratio is 62:1, the maximum temperature of gases at the H.P. turbine inlet is 1000 K. Take C_p of air as 1.005 and C_p for gases as 1.11 kJ/kgK. The heating value of fuel is 40 mJ/kg.

(*Ans.* 698.53 kW, 0.318, 21.65 %)

21.4 Air enters the compressor of a gas turbine equipped with a regenerator at 1 bar, 300 K. The pressure ratio of the compressor is 6.6, its efficiency is 0.82. The regenerator effectiveness is 0.85, the combustion chamber efficiency is 0.8 and the turbine efficiency is 0.85. The pressure drop in the regenerator on the air and gas side is 0.06 and 0.05 bar respectively. The heating value of the fuel is 40 mJ/kg and the air – fuel ratio is 65:1. Estimate the efficiency of the plant with and without a regenerator.

(*Ans.* 28.95 % and 19.15 %, with and without regenerator.)

21.5 In a closed-cycle gas turbine cycle, helium at 15 bar, 300 K enters the compressor ($\eta_c = 0.8$) and comes out at 60 bar. The gas is heated in a heat exchanger to 1000 K before entering the turbine ($\eta_T = 0.85$). The exhaust from the turbine is cooled to the initial temperature in a cooler. Calculate the cycle efficiency and the net output for a flow rate of 75 kg/s. Take $C_p = 5$ kJ/kg K and $\gamma = 1.5$. (*Ans.* 19.6 %, 35.54 mw.)

21.6 A simple open-cycle gas turbine plant works between the pressures of 1 bar and 6 bar and temperatures of 300 K and 1023 K. The calorific value of fuel is 44 MJ/kg. If the mechanical efficiency and the generator efficiency are 95% and 96% respectively, and for an air-flow rate of 20 kg/s, calculate (a) the air-fuel ratio, (b) the thermal efficiency, and (c) the power output.

21.7 A turbojet aircraft is travelling at 925 km/h in atmospheric conditions of 0.45 bar and –26°C. The compressor pressure ratio is 8, the air mass flow rate is 45 kg/s and the maximum allowable cycle temperature is 800°C. The compressor, turbine and jet pipe stagnation isentropic efficiencies are 0.85, 0.89 and 0.9 respectively, the mechanical efficiency of the drive is 0.98 and the combustion efficiency is 0.99. Assuming a convergent propulsion nozzle, loss of stagnation pressure in the combustion chamber of 0.2 bar, and a fuel of calorific value of 43.3 MJ/kg, calculate (a) the required nozzle exit area, (b) the net thrust developed, (c) the air–fuel ratio, and (d) the specific fuel consumption. For gases in the turbine and propulsion nozzle, take $\gamma = 1.333$ and $C_p = 1.15$ kJ/kgK.

Ans. ((a) 0.216 m^2 (b) 19.94 kN, (c) 70.87, (d) 0.0319 kg/kNs)

21.8 In a turboprop engine, the compressor pressure ratio is 6 and the maximum cycle temperature is 760°C. The stagnation isentropic efficiencies of the compressor and turbine are 0.85 and 0.88 respectively, the mechanical efficiency is 0.99. The aircraft is flying at 725 km/h at an altitude where the ambient temperature is –7°C. Taking an intake duct efficiency of 0.9, neglecting and pressure loss in the combustion chamber, and assuming that the gases in the turbine expand down to atmospheric pressure, leaving the aircraft at 725 km/h relative to the aircraft, calculate (a) the specific power output, and (b) the cycle efficiency. For the gases in the turbine, take $\gamma = 1.333$ and $c_p = 1.15$ kJ/kgK.

(*Ans.* (a) 170.2 kW per kg/s (b) 28.4%)

21.9 Afterburning is used in the aircraft of Problem 21.8 to obtain an increase in thrust. The stagnation temperature after the afterburner is 700°C and the pressure loss in the afterburning process is 0.07 bar. Calculate the nozzle exit area now required to pass the same mass flow rate as in Problem 21.8 and the new net thrust.

(*Ans.* 0.24 m^2, 22 kN.)

21.10 A turbojet unit flies at 225 m/s in air where $\phi = 0.62$ bar and T = 265 K. The unit has diffuser ($\eta = 0.9$), compressor ($\eta = 0.85$, pressure ratio 6), combustion chamber (pressure loss 0.15 bar, $\eta = 0.9$), turbine ($\eta = 0.88$, inlet temperature 1100 K, and nozzle ($\eta = 0.9$) Calculate the thrust and the air–fuel ratio, if the heating value of the fuel is 40 MJ/kg. (*Ans.* 392.32 N/kg air, 59.73.)

21.11 The drag for a turbojet aircraft is 7 kN and the propulsive efficiency is 0.55. When the aircraft is flying with a speed of 800 km/h, estimate (a) the diameter of the jet if the air density is 0.17 kg m^3, (b) the air–fuel ratio when the overall efficiency of the unit is 20% and (c) the specific fuel consumption if the heating value of the fuel is 40 MJ/kg.

(*Ans.* (a) 4.96 cm, (b) 98.97, (c) 0.45 kg/kWh)

21.12 A propeller of 3.5 m, diameter-produces a thrust of 20 kN while flying at a speed of 100 m/s where the pressure is 1 bar and the temperature is 280 K. Estimate the velocity in the final wake, the pressure rise through the propellerer, and the propeller efficiency.

(*Ans.* $V_4 = 115.5$ m/s, $\Delta p = 2.0786$ kPa, $\eta_p = 92.8\%$)

21.13 An aircraft is flying at a speed of 250 m/s and the air mass flow rate is 45 kg/s. The air–fuel ratio is 60 and the heating value is 40 MJ/kg. If the gases expand to ambient pressure in the nozzle, determine (a) the jet velocity when thrust power is maximum, (b) the thrust and thrust power, and (c) the propulsive and overall efficiency of the system.

(*Ans.* (a) 500 m/s, (b) 11.625 kN and 2.91 Mw, (c) 0.674 and 0.0968.)

21.14 The combustion gases from a rocket motor have negligible velocity when leaving the CC at a pressure of 25 bar and a temperature of 3000 K. The gases expand through a convergent–divergent nozzle to a pressure of 1.5 bar in the exit plane of the nozzle. Assuming isentropic expansion and taking for gases $\gamma = 1.2$ and a motor mass of 32 kg/kgmol and neglecting dissociation effects, calculate the ratio of exit area A_e to throat area A_t. Also, calculate the thrust developed per unit throat area when the rocket is operating in outer space.

Ans. 3.19, 11.415 MN/m^2 throat area.

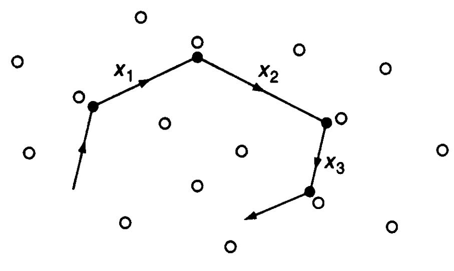
C H A P T E R

22 Transport Processes in Gases

Collisions between molecules were not considered in Chapter 21 while deriving the expressions for pressure and temperature of an ideal gas in terms of its molecular properties. Intermolecular collisions will now be considered.

22.1 MEAN FREE PATH AND COLLISION CROSS-SECTION

Let us single out one particular molecule represented by the black circle and trace its path among the other molecules, which would be assumed to be frozen in their respective positions (Fig. 22.1). The distance traversed by a molecule between successive collisions is called the free path, denoted by x, and the average length of these paths is called the *mean free path*, denoted by λ. The molecules are assumed to be perfectly elastic spheres of radius r. As two molecules collide, the centre-to-centre distance is $2r$, which would remain the same if the radius of the moving molecule is increased to $2r$ and the stationary molecules are shrunk to geometrical points, as shown in Fig. 22.1 The cross-sectional area of the moving molecule is called the *collision cross-section* σ, and it is given by $\sigma = 4\,\pi r^2$



Free paths a gas molecule

The moving molecule sweeps out in time t, a cylindrical volume of cross-sectional area σ and length $\bar{v}t$, where $\bar{v}$ is the average velocity of the molecule. The number of collisions it makes during this time, will be the same as the number of molecules whose centres lie within this volume, which is $\sigma n \bar{v} t$, where n is the number of molecules per unit volume. The number of collisions per unit time is known as the *collision frequency*, denoted by z, which is:

$$z = \sigma n \bar{v} \tag{22.1}$$

The mean free path of the molecules is given by:

$$\lambda = \frac{\text{Distance travelled in time } t}{\text{Number of collisions in time } t} = \frac{\bar{v}t}{\sigma n \bar{v} t} = \frac{1}{\sigma n} \tag{22.2}$$

On an average, the diameter (d) of the molecules is (2 to 3) $\times 10^{-10}$ m, the distance between molecules 3×10^{-9} m (or 10d), and the mean free path is about 3×10^{-8} m (or $100d$).

If motion of all the molecules if considered and all the molecules move with the same speed, a correction is required and λ is obtained as:

$$\lambda = 0.75/\sigma n \tag{22.3}$$

If the Maxwellian velocity distribution is assumed for the molecules, $\lambda = 0.707/\sigma n$ (22.4)

For an electron moving among molecules of a gas, the radius of the electron is so small compared to that of a molecule that in a collision the electron may be treated as a point and the centre-to-centre distance becomes

r, instead of $2r$, where r is radius of the molecule. Also, the velocity of the electron is so much greater than the velocities of the molecules that the latter can be considered stationary. As a result, no correction is required, and the electronic mean free path λ_e is given by:

$$\lambda_e = 4/\sigma n \tag{22.5}$$

where

$$\sigma = 4\pi r^2$$

22.2 DISTRIBUTION OF FREE PATHS

The distance travelled by a molecule between successive collisions or the free path x varies widely. It may be greater or less than λ, or equal to it. Just like distribution of molecular velocities, we will now determine how many molecules will have free paths in a certain range, say between x and $x + dx$.

Let us consider a large number of molecules N_0 at a certain instant (Fig. 22.2). If the molecules collide, they will be assumed to get removed from the group. Let N represent the number of molecules left in the group after travelling a distance x. Then these N molecules have free paths larger than x. In the next short distance dx, let dN number of molecules make collisions and get removed from the group. So, these dN molecules which have free paths lying between x and $x + dx$ are proportional to N and to dx. Since N is always decreasing, dN is negative and it is given by:

$$dN = -P_c N\,dx \tag{22.6}$$

where P_c is the constant of proportionality, known as the *collision probability*.

Then

$$dN/N = -P_c\,dx$$

$$\therefore \quad \ln N = -P_c x + A$$

Where A is constant. When $x = 0$, $N = N_0$, and so $A = \ln N_0$. Therefore,

$$N = N_0 e^{-P_c x} \tag{22.7}$$

The number of molecules that remains in the group falls off exponentially with x. From Eq. (22.6),

$$dN = -P_c N_0 e^{-P_c x}\,dx \tag{22.8}$$

Using this expression for dN, the mean free path λ becomes

$$\lambda = \frac{\int x\,dN}{\int dN} = \frac{\int_0^\infty -x P_c N_0 e^{-P_c x}\,dx}{N_0} = \frac{1}{P_c}$$

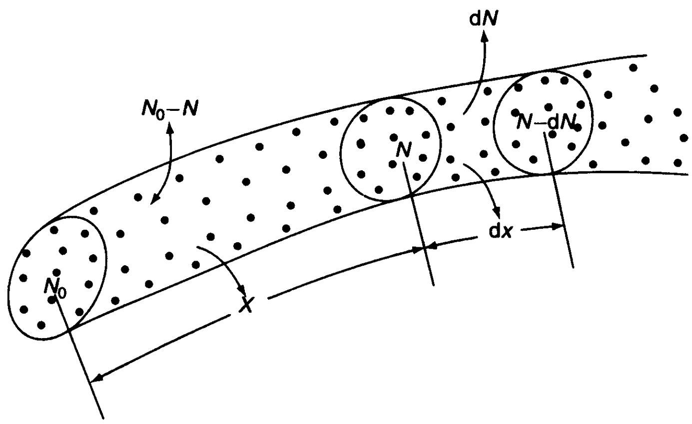


Fig. 22.2 Molecules colliding and getting removed from the group

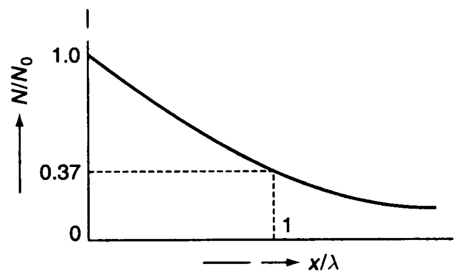


Fig. 22.3 Plot of survival equation

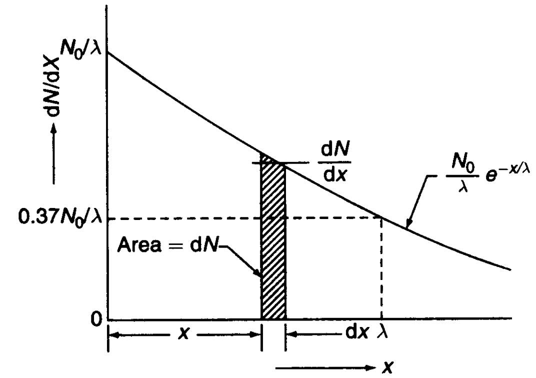


Fig. 22.4 Distribution of free paths

Since $\lambda = 1/\sigma n$, $P_c = \sigma n$. The collision probability is thus proportional to the collision cross-section and the number of molecules per unit volume. The Eq. (22.7) can thus be written as

$$N = N_0 e^{-x/\lambda} \tag{22.9}$$

It is know as the *survival equation* which indicates the number of molecules N, out of N_0, which survive collision and have free paths longer than x. A plot of N/N_0 vs x/λ is shown in Fig. 22.3. If $x/\lambda = 1$, ie., $x = \lambda$, $N/N_0 = 0.37$. The fraction of free paths longer than λ is, therefore, 37% and the fraction shorter than λ is 63%.

Differentiating Eq. (22.9) $$dN = \frac{N_0}{\lambda} e^{-x/\lambda} dx$$

or, $$dN/dx = -\frac{N_0}{\lambda} e^{-x/\lambda} \tag{22.10}$$

This equation represents the distribution of free paths. It is plotted in Fig. 22.4. The area of the narrow vertical strip of thickness dx at a distance x from the origin represents dN, the number of molecules with free paths of lengths between x and x + dx.

22.3 TRANSPORT PROPERTIES

A Simple treatment based on the concept of the mean free path will now be given for four transport properties of a gas, viz., coefficient of viscosity, thermal conductivity, coefficient of diffusion and electrical conductivity, which govern respectively the transport of momentum, energy, mass, and electric charge within the gas by molecular motion.

22.3.1 Coefficient of Viscosity

Let us consider a gas flowing over a flat stationary plate. Due to viscous effect there is the growth of a boundary layer over the plate surface.

The velocity of fluid at the surface will be zero, and it gradually increases to free stream velocity as shown in Fig. 22.5, drawn for laminar flow. Let us imagine a surface P-P within the gas at an arbitrary height from the plate, where the fluid velocity is u and the velocity gradient du/dy. The velocity u is superposed on the random thermal motion of the molecules.

Let us consider a volume element dV at a distance r from an element of area dA in the plane P-P, making an angle θ with normal to dA (Fig. 22.6), the plane P-P being the same as shown in Fig. 22.5. The volume

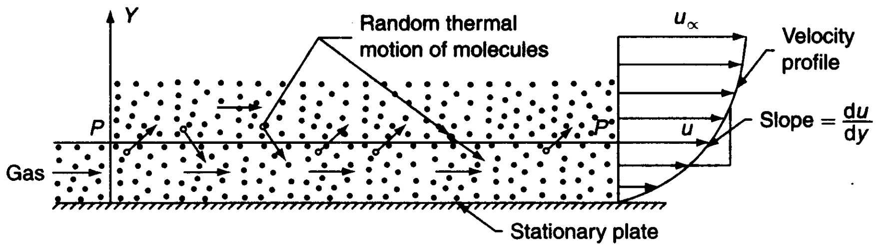


Fig. 22.5 *Flow of a gas over a flat plate*

element is very small when compared with the physical dimensions of the system, but large enough to contain many molecules. The total number of molecules in dV is ndV, and the total number of collisions within dV in time dt is $\frac{1}{2}Zn\cdot$ d$V \cdot$ dt, where z is the collision frequency of a molecule, n is the number of molecules per unit volume, and the factor $\frac{1}{2}$ is required since two molecules are involved in each collision. Since two new free paths originate at each collision, the total number of new free paths, or molecules, originating in dV is ZndVdt. If we assume that these molecules are uniformly distributed in direction throughout the solid angle 4π, then the number headed towards the elemental area dA is:

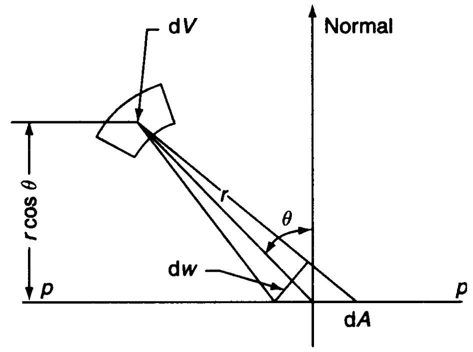


Fig. 22.6 *Transfer of momentum across the plane P-P by molecules in random thermal motion*

$$\frac{zn\mathrm{d}V\,\mathrm{d}t}{4\pi}\mathrm{d}w$$

where dw is the solid angle subtended at the centre of dV by the area dA and is equal to (dA cos θ)/r^2.

The number of molecules that leave dV and reach dA without having made a collision may be found from the survival equation, Eq. (22.9), as given below:

$$\frac{zn\,\mathrm{d}V\mathrm{d}t}{4\pi}\mathrm{d}w\,e^{-r/\lambda}$$

Since d$V = r^2 \sin\theta \cdot \mathrm{d}\theta \cdot \mathrm{d}\phi \cdot \mathrm{d}r$, the number of molecules leaving dV in time dt and crossing dA without any collision is

$$\frac{1}{4\pi}\frac{\mathrm{d}A\cos\theta}{r^2}Zn\,r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}r\,\mathrm{d}t\,e^{-r/\lambda}$$

The total number of molecules crossing dA in time dt from the top is

$$N_{\text{total}} = \frac{Zn\,\mathrm{d}A\,\mathrm{d}t}{4\pi}\int_{\theta=0}^{\pi/2}\sin\theta\cos\theta\,\mathrm{d}\theta\int_{\phi=0}^{2\pi}\mathrm{d}\phi\int_{r=0}^{\infty}e^{-r/\lambda}\mathrm{d}r$$

$$= \frac{Zn\,\mathrm{d}A\,\mathrm{d}t}{4\pi}\frac{1}{2}2\pi\cdot\lambda = \frac{1}{4}Zn\,\lambda\,\mathrm{d}A\,\mathrm{d}t \tag{22.11}$$

Since the dimensions of the physical system are very much larger than the molecular free path, the integral over r has been extended to infinity.

But $z = \bar{v}/\lambda$, so the number of molecules crossing the plane P-P from the top (or bottom) per unit area and per unit time is $\frac{1}{4}n\bar{v}$. This is the same result obtained earlier in Sec. 21.4 without considering any intermolecular collision.

These molecules crossing the plane *P-P* may be visualized as carrying properties characteristic of an average distance $\bar{y}$, either above or below the plane at which they made their last collisions before crossing. To find $\bar{y}$, each molecule crossing from dV is multiplied by its distance $r \cos\theta$ from the *P-P* plane, it is integrated over θ, ϕ and r and then divided by N_{total} crossing the plane.

$$\bar{y} = \frac{\int y\, dN}{\int dN} = \frac{\int r\cos\theta\, dN}{N_{total}}$$

$$= \frac{1}{N_{total}}\left[\int_r \int_\phi \int_\theta \frac{1}{4\pi} Znd\,A\,dt.\,e^{-r/\lambda}\sin\theta.\cos\theta.\,d\theta\,d\phi\,dr\,r\cos\theta\right]$$

$$= \frac{1}{N_{total}}\left[\frac{ZndA\,dt}{4\pi}\int_{\theta=0}^{\pi/2}\sin\theta\cos^2\theta\,d\theta\int_{\phi=0}^{2\pi}d\phi\int_{r=0}^{\infty}re^{-r/\lambda}dr\right]$$

$$= \frac{\dfrac{Zn\,dA\,dt}{4\pi}\dfrac{1}{3}2\pi\cdot\lambda^2}{\dfrac{1}{4}Zn\,\lambda\,dA\,dt} = \frac{2}{3}\lambda \tag{22.12}$$

The velocity of gas at a height $\bar{y}$ above *PP* is,

$$u + \frac{2}{3}\lambda\frac{du}{dy}$$

if the velocity gradient is considered constant over distances of the order of a free path.

The net momentum in the direction of flow carried across the plane by the molecules crossing *PP* from above per unit area and per unit time is:

$$\frac{1}{4}n\,\bar{v}\,m\left[u + \frac{2}{3}\lambda\frac{du}{dy}\right]$$

Similarly, the net momentum transfer from below is:

$$\frac{1}{4}n\,\bar{v}\,m\left[u - \frac{2}{3}\lambda\frac{du}{dy}\right]$$

The difference between the above two quantities is the net rate of transport of momentum per unit area and per unit time, given by:

$$\frac{1}{3}nm\bar{v}\lambda\frac{du}{dy}$$

From Newton's law of viscosity, this is the viscous force per unit area $\tau = \mu\dfrac{du}{dy}$, where μ is the coefficient of viscosity.

Therefore,

$$\mu = \frac{1}{3}nm\bar{v}\lambda \tag{22.13}$$

Putting $\sigma = 1/\lambda n$ from Eq. (22.2),

$$\mu = \frac{1}{3}\frac{m\bar{v}}{\sigma} \tag{22.14}$$

Where σ is the collision cross-section.

For a gas with a Maxwellian velocity distribution, $\bar{v} = [8\,KT/\pi m]^{1/2}$, $\lambda = 0.707/\sigma n$

Therefore, from Eq. (22.13),
$$\mu = \frac{2}{3\sigma}[mKT/\pi]^{1/2} \tag{22.15}$$
Putting
$$\sigma = \pi d^2$$
$$\mu = \frac{2}{3\pi^{3/2}}\frac{(mKT)^{1/2}}{d^2} \tag{22.16}$$

A significant conclusion from this equation is that the viscosity of a gas is independent of the pressure or the density and *depends only on the temperature*. As the temperature of the gas increases, its viscosity increases.

22.3.2 Thermal Conductivity

It is possible to derive an expression for the thermal conductivity of a gas by repeating the argument leading up to the Eq. (22.12). For a system, not being isothermal. the molecules moving from the warmer region to the colder region carry with them more energy than those moving in the opposite direction, resulting in a net transfer of energy.

Let us consider a gas confined between two stationary plates (Fig. 22.7) maintained at different temperatures. Let T be the temperature at the plane P-P and dT/dY the temperature gradient. The mean energy of a molecule at a temperature T is given by $\frac{f}{2}KT$, where f is the number of degrees of freedom. It was concluded from Eq. (22.12) that the molecules crossing the plane from either direction carry property values characteristic of the planes $(2/3)\lambda$ distance from the plane.

Therefore, the energy carried across the plane per unit area and per unit time, by the molecules crossing the plane from above:
$$\frac{1}{4}n\bar{v}\frac{f}{2}K\left[T+\frac{2}{3}\lambda\frac{dT}{dy}\right]$$
and the energy carried by the molecules crossing from below:
$$\frac{1}{4}n\bar{v}\frac{f}{2}K\left[T-\frac{2}{3}\lambda\frac{dT}{dy}\right]$$

The net rate energy transfer per unit area is the difference of the above two quantities, which gives:
$$\frac{1}{6}n\bar{v}fK\lambda\frac{dT}{dy}$$
By Fourier's law, this is given by:
$$q = K\frac{dT}{dy}$$

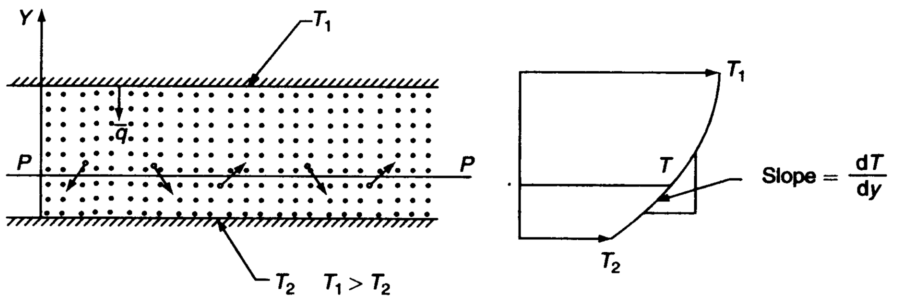


Fig. 22.7 Transfer of energy in a gas

where k is the thermal conductivity of the gas. Therefore,

$$k = \frac{1}{6} n \bar{v} f K \lambda \tag{22.17}$$

or,

$$k = \frac{1}{6} \frac{\bar{v} f K}{\sigma} \tag{22.18}$$

For a gas with a Maxwellian velocity distribution,

$$\bar{v} = [8KT/\pi m]^{1/2} \text{ and } \lambda = 0.707/\sigma n$$

$$k = \frac{1}{3} \frac{f K}{\sigma} [KT/\pi m]^{1/2} \tag{22.19}$$

The above equation predicts that the thermal conductivity of a gas, like the viscosity, is independent of pressure or density, and *depends only on temperature*. It increases as the temperature increases.

For a monatomic gas, $f = 3$ and putting $\sigma = \pi d^2$,

$$k = \frac{1}{\pi^{3/2} d^2} \left[\frac{K^3 T}{m} \right]^{1/2}$$

Dividing Eq. (22.15) by Eq. (22.19),

$$\mu / k = 2m / fK \tag{22.20}$$

But

$$m = M/N_0,\ K = \bar{R}/N_0,\ c_v = \frac{f}{2} R = \frac{f}{2} \frac{\bar{R}}{M},$$

where M is the molecular weight and N_0 is the Avogadro's number. Therefore, on substitution in Eq. (22.20)

$$\mu c_v / k = 1 \tag{22.21}$$

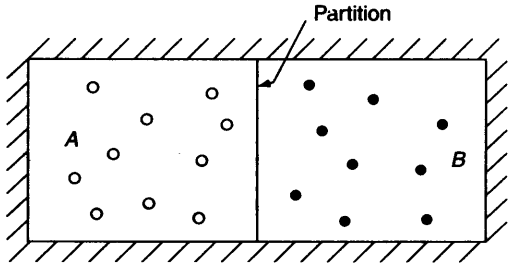
or, Prandtl number,

$$Pr = \mu c_p / k = \gamma \tag{22.22}$$

The results given by the Eqs (22.21) and (22.22) agree with the experimental values only as regards order of magnitude.

22.3.3 Coefficient of Diffusion

In a gaseous mixture, diffusion results from random molecular motion whenever there is a concentration gradient of any molecular species. Let us consider two different gases A and B at the same temperature and pressure on the two opposite sides of the partition in a vessel (Fig. 22.8). The number of molecules per unit volume (p/KT) is, therefore, the same on both sides. When the partition is removed, both the gases diffuse into each other, and after a lapse of time both the gases are uniformly distributed throughout the entire volume. The diffusion process is often superposed by the hydrodynamic flow resulting from pressure differences, and the effects of molecules rebounding from the walls of the vessel. When more than one type of molecule is present, the rates of diffusion of one species into another are different. To simplify the problem we assume: (1) the molecules of a single species diffusing into others of the same species (self-diffusion), (2) the containing vessel very large compared with the mean free path so that collisions with the walls can be neglected in comparison with collisions



Diffusion of gases A and B when the partition is removed with other molecules, and (3) a uniform pressure maintained so that there is no hydrodynamic flow. Of course, if all the molecules are exactly alike, there would be no way experimentally to identify the diffusion process. However, the diffusion of molecules that are isotopes of the same element is a practical example of the self-diffusion process.

Let n denote the number of molecules per unit volume of one gas, blackened for identification (Fig. 22.9). Let us consider diffusion across an imaginary vertical plane y-y in the vessel. Let us also assume that n increases from left to right in the positive x-direction. The number of black molecules crossing the plane from right to left exceeds the number crossing in the opposite direction. The net transfer of black molecules from left to right per unit area and per unit time, denoted by Γ, is given by:

$$\Gamma = -D\frac{dn}{dx} \tag{22.23}$$

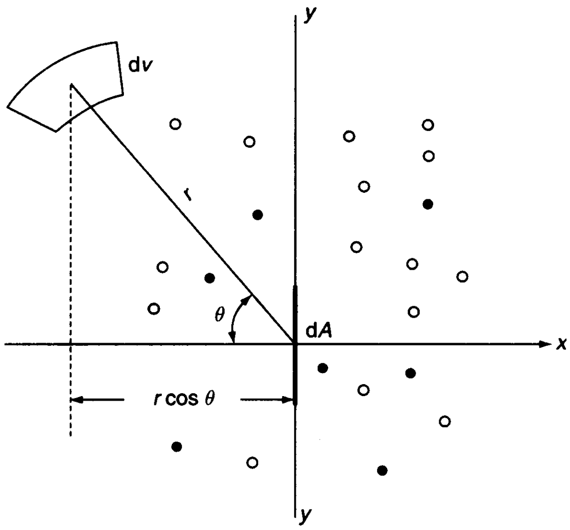


Fig. 22.9 *Diffusion across an imaginary plane*

where D is the coefficient of diffusion.

Let us consider a volume element dV at a distance r from an element of area dA in the plane y-y, making an angle θ with normal to dA. The concentration of black molecules in the plane of dV at a distance x from the vertical plane,

$$n = n_0 + x\frac{dn}{dx} = n_0 - r\cos\theta\frac{dn}{dx} \tag{22.24}$$

Where n_0 is the concentration at the vertical plane. The total number of free paths originating in dV in time dt is $Zn_t dVdt$, where n_t is the total number of molecules per unit volume. The number of free paths of black molecules will be $\frac{n}{n_t}(Zn_t dVdt)$, or $ZndVdt$. The number of black molecules crossing dA without making a collision, as found in Section 22.3.1 is $\frac{1}{4\pi}Zn\, dA\, dt \sin\theta\cos\theta\, e^{-r/\lambda}\, d\theta\, d\phi\, dr$. By substituting for n from Eq. (22.24), it is:

$$\frac{1}{4\pi}Zn_0\, dA\, dt \sin\theta\cos\theta\, e^{-r/\lambda}d\theta\, d\phi\, dr$$

$$-\frac{1}{4\pi}Z\frac{dn}{dx}dA\, dt \sin\theta\cos^2\theta\, r\, e^{-r/\lambda}d\theta\, d\phi\, dr$$

Integrating the above expressions over θ from 0 to $\pi/2$, over ϕ from 0 to 2π and over r from 0 to ∞, the number becomes

$$\frac{1}{4}Z\, n_0\lambda\, dA\, dt - \frac{1}{6}Z\lambda^2\frac{dn}{dx}dA\, dt$$

Therefore, the number of black molecules crossing the plane y-y from left to right per unit area and per unit time is:

$$\underset{\rightarrow}{\Gamma} = \frac{1}{4}Z\, n_0\lambda - \frac{1}{6}Z\lambda^2\frac{dn}{dx} \tag{22.25}$$

Similarly, the number crossing from right to left is

$$\underset{\leftarrow}{\Gamma} = \frac{1}{4}Z\, n_0\lambda + \frac{1}{6}Z\,\lambda^2\frac{dn}{dx} \tag{22.26}$$

By subtracting Eq. (22.26) from Eq. (22.25), the net number crossing from left to right becomes

$$\Gamma = -\frac{1}{3} Z \lambda^2 \frac{dn}{dx} \tag{22.27}$$

From Eqs (22.23) and (22.27)

$$D = \frac{1}{3} Z\lambda^2 \tag{22.28}$$

Putting $z = \bar{v}/\lambda$,

$$D = \frac{1}{3}\bar{v}\lambda \tag{22.29}$$

For Maxwellian velocity distribution of the molecules,

$$\bar{v} = [8KT/\pi m]^{1/2} \quad \text{and} \quad \lambda = 0.707/\sigma n_t$$

$$\therefore \qquad D = \frac{2}{3\sigma n_t}[KT/\pi m]^{1/2} \tag{22.30}$$

or,

$$D = \frac{2}{3\pi^{3/2} d^2 n_t}[KT/m]^{1/2} \tag{22.31}$$

The equation applies to diffusion in a binary mixture of almost identical gases. Dividing Eq. (22.15) by Eq. (22.30),

$$\mu/D = n_t . m = \rho$$

or, Schmidt number

$$Sc = \mu/\rho D = 1 \tag{22.32}$$

Measured values of Schmidt number for the diffusion of isotopic tracer molecules yield values between 1.3 and 1.5, which indicate qualitative agreement of theory with measured data.

22.3.4 Electrical Conductivity

Conduction of electricity in a gas arises as a result of motion of the free electrons present in the gas. When high-energy atoms in the gas collide, some collisions cause ionization when an electron is separated from its atom, so that a negatively charged electron and a positively charged ion are produced from the neutral atom. Most gases at room temperature do not have many such high-energy molecules and thus have very few free electrons. At high temperatures, however, an appreciable number of electrons may be liberated and the gases may become highly conductive. In absence of external electrical fields, the electrons will be distributed uniformly throughout the gas volume. As an electrical field is impressed on the gas, the charged particles are accelerated with a force

$$F = q_e E = m_e \frac{dv_e}{dt} \tag{22.33}$$

where q_e is the charge on the electron and E is the electric field strength (volts per metre). Integrating Eq. (22.33)

$$v_e = \frac{dx_e}{dt} = \frac{q_e Et}{m_e}$$

At $t = 0$, $v_e = 0$, $x_e = 0$. Similarly, the velocity of a single charged ion is: $v_i = \dfrac{q_i Et}{m_i}$ (22.34)

where q_i is the charge on an ion and m_i is the mass of the ion. Since, $m_i/m_e = 1840\,M$, where M is the atomic weight of the ion, $v_i/v_e = 1/1840\,M$. Therefore, the velocity of the ions is very small compared to the velocity of electrons and is neglected.

The electron velocity v_e is randomly oriented in absence of an electric field with no preferred direction of motion. The electron would attain the velocity v_e (in Eq. 22.33) after t seconds of impressing the electric field, assuming it had to collision. To consider the effects of collision, it is assumed that the electrons are brought to rest after each collision, reaccelerated by the electric field, and then undergo another collision. The distance traveled between collision x_e is obtained by integrating Eq. (22.33).

$$x_e = (q_e E t^2)/2m_e$$

where t is the collision time.

The average distance travelled between collisions is

$$\bar{x}_e = \frac{1}{N_e}\int x_e \mathrm{d}N_e$$

where N_e is the total number of electrons in the gas.

The distribution of collision times is identical to distribution of free paths given by Eq. (22.10), which is

$$\mathrm{d}N_e = \frac{N_e}{\tau_e} e^{-t/\tau_e}\mathrm{d}t \tag{22.35}$$

where τ_e is the mean collision time for electrons.

The average distance travelled in the electric field by the electrons between collisions, $\bar{x}_e$, divided by the mean collision time τ_e is called the *electron drift velocity*, v_{e_d}. Therefore

$$\begin{aligned} v_{e_d} &= \bar{x}_e/\tau_e = \frac{1}{\tau_e}\frac{1}{N_e}\int x_e \mathrm{d}N_e \\ &= \frac{1}{\tau_e}\frac{1}{N_e}\int_0^\infty \frac{q_e E t^2}{2m_e}\frac{N_e}{\tau_e} e^{-t/\tau_e}\mathrm{d}t \\ &= \frac{1}{\tau_e^2}\frac{q_e E}{2m_e}\int_0^\infty t^2 e^{-t/\tau_e}\mathrm{d}t = \frac{q_e E \tau_e}{m_e} \end{aligned} \tag{22.36}$$

From Eq. (22.5), the electronic mean free path is

$$\lambda_e = 4/\sigma n$$

Also, $\lambda_e = \bar{v}_e \tau_e$, where $\bar{v}_e$ is the average velocity of the electrons, given by $[8KT/\pi m_e]^{1/2}$ for Maxwellian velocity distribution. Therefore,

$$v_{e_d} = \frac{q_e E}{m_e}\frac{4}{\sigma n}\left[\frac{\pi m_e}{8KT}\right]^{1/2}$$

Using

$$n = p/KT,$$

$$u_{e_d} = \frac{4 q_e E}{\sigma p}\left[\frac{\pi KT}{8 m_e}\right]^{1/2} \tag{22.37}$$

where σ is the atomic cross-section.

The flux of charge across unit area per unit time is called the *current density*, *J*. The current density is defined with respect to the average drift velocity by the following equation,

$$J = n_e q_e v_{e_d} \tag{22.38}$$

where n_e is the number density of electrons. The motion of the ions is neglected. From Eqs (22.36) and (22.38),

$$J = \frac{q_e^2 n_e \tau_e}{m_e} E \tag{22.39}$$

The current density is proportional, to the electric field and the constant of proportionality is called the *electrical conductivity*, σ_e $\quad J = \sigma_e E$

From Eq. (22.39), σ_e is given by

$$\sigma_e = \frac{q_e^2 n_e \tau_e}{m_e}$$

Putting

$$\lambda_e = \bar{v}\,\tau_e = 4/\sigma n,$$

$$\sigma_e = \frac{q_e^2 n_e}{m_e}\frac{4}{\sigma n}\left[\frac{\pi m_e}{8KT}\right]^{1/2}$$

$$= \frac{q_e^2 n_e}{\sigma n}\left[\frac{2\pi}{m_e KT}\right]^{1/2} \tag{22.40}$$

The electron drift velocity v_{e_d} is also proportional to the electric field and the proportionality constant is called the electron mobility, μ_e

$$v_{e_d} = \mu_e E$$

From Eq. (22.36)

$$\mu_e = (q_e/\tau_e)/m_e$$

Again, using

$$\lambda_e = \bar{v}_e\,\tau_e = 4/\sigma n,$$

$$\mu_e = \frac{q_e}{m_e}\frac{4}{\sigma n}\left[\frac{\pi m_e}{8KT}\right]^{1/2} = \frac{q_e}{\sigma n}\left[\frac{2\pi}{m_e KT}\right]^{1/2} \tag{22.41}$$

Solved Examples

Example 22.1

Calcuate the mean free path for oxygen molecules at 1 atm pressure and 300 K. What fraction of molecules have free paths longer than 2λ?. The effective diameter of the oxygen molecule is 3.5 Angstroms.

Solution The collision cross section of the oxygen molecule is:

$$\sigma = \pi d^2 = \pi(3.5\times10^{-10})^2 = 3.84\times10^{-19}\,\text{m}^2$$

Number of molecules per unit volume

$$n = p/KT = (1.013\times10^5)/(1.38\times10^{-23}\times300)$$

$$= 2.45\times10^{25}\,\text{molecules/m}^3$$

The mean free path is given by Eq. (22.4)

$$\lambda = 0.707/\sigma n = 0.707/(3.84\times10^{-19}\times2.45\times10^{25})$$

$$= 7.25\times10^{-8}\,\text{m}$$ *Ans.*

From the Eq. (22.9), putting $x = 2\lambda$

$$N/N_0 = e^{-x/\lambda} = e^{-2} = 0.135$$

i.e., 13.5% of the molecules have free paths greater than 2λ.

Example 22.2

The mean free path of the molecules of a certain gas at a temperature of 25°C is 2.63×10^{-5} m. The radius of the molecules is 2.56×10^{-10} m.
(a) Find the pressure of the gas. (b) Calculate the number of collisions made by a molecule per metre of path.

Solution The collision cross-section, $\sigma = 4\pi r^2 = 4\pi \times (2.56 \times 10^{-10})^2 = 82.36 \times 10^{-20}\,\text{m}^2$

Number of molecules per unit volume of the gas is:

$$n = \frac{0.707}{\sigma\lambda} \text{ (assuming Maxwell-Boltzmann velocity distribution)}$$

$$= \frac{0.707}{82.36 \times 10^{-20} \times 2.63 \times 10^{-5}} = 0.326 \times 10^{23}\,\text{molecules m}^3$$

From the ideal gas equation of state,

$$P = nKT$$

$$= 0.326 \times 10^{23}\,\frac{\text{molecules}}{m^3} \times 1.38 \times 10^{-23}\,\frac{\text{J}}{\text{molecule - K}} \times 298\,\text{K}$$

$$= 0.326 \times 1.38 \times 298\,\text{N/m}^2 = 134.06\,\text{Pa}$$ *Ans.* (a)

Number of collisions made by a molecule:

$$= \frac{1}{\lambda} = \frac{1}{2.63 \times 10^{-5}} = 3.8 \times 10^4$$ *Ans.* (b)

Example 22.3

The mean free path in a certain gas is 10 cm. If there are 10,000 free paths, how many are longer than (a) 10 cm, (b) 20 cm, (c) 50 cm, (d) How many are longer than 5 cm, but shorter than 10 cm. (e) How many are between 9.5 cm and 10.5 cm in length? (f) How many are between 9.9 cm and 10.1 cm in length? (g) How many are exactly 10 cm in length?

Solution From the survival equation, $N = N_0 e^{-x/\lambda}$

where $N_0 = 10{,}000$ and $\lambda = 10$ cm (given)

$$N = 10{,}000\, e^{-x/10}$$

is the governing equation,

(a) For $x > 10$ cm, $N = 10{,}000\, e^{-1} = 10{,}000/2.718 = 3680$ *Ans.*

(b) For $x > 20$ cm, $N = 10{,}000\, e^{-2} = 1354$ *Ans.*

(c) For $x > 50$ cm, $N = 10{,}000\, e^{-5} = 67.4$ or 68 *Ans.*

(d) Fox x between 5 cm and 10 cm, we have

$$dN = \frac{N_0}{\lambda} e^{-x/\lambda} dx$$

$$N = \int_5^{10} -\frac{N_0}{\lambda} e^{-x/\lambda} dx = -\frac{N_0}{\lambda}[e^{-x/10}(-\lambda)]_5^{10}$$

$$= -N_0[e^{-x/10}]_5^{10} = N_0[e^{-1} - e^{-0.5}]$$
$$= N_0[0.3679 - 0.6066] = -0.2387 \times 10,000$$
$$= -2387$$ *Ans.*

The negative sign implies than N is decreasing with x.

(e) For x between 9.5 cm and 10.5 cm

$$N = -10,000[e^{-9.5/10} - e^{-10.5/10}]$$
$$= -10,000(0.3870 - 0.3500) = -370$$ *Ans.*

(f) For x between 9.9 cm and 10.1 cm

$$N = -10,000[e^{-9.9/10} - e^{-10.1/10}]$$
$$= -10,000(0.3716 - 0.3642) = -74$$ *Ans.*

(g) For $x = 10$ cm exactly

$$N = -\frac{N_0}{\lambda}\int_{10}^{10} e^{-x/\lambda}dx = \text{zero}$$ *Ans.*

Example 22.4

Calculate the coefficient of viscosity of oxygen at 1 *atm pressure and* 300 *K.*

Solution From previous examples, we have

$$m = 5.31 \times 10^{-26} \text{ kg/molecule}$$
$$\bar{v} = 445 \text{ m/s}$$
$$\sigma = 3.84 \times 10^{-19} \text{ m}^2$$

Therefore,

$$\mu = \frac{1}{3}\frac{m\bar{v}}{\sigma}$$
$$= \frac{(5.31 \times 10^{-26}) \text{ kg/molecule} \times 445 \text{ m/s}}{3 \times 3.84 \times 10^{-19} \text{m}^2\text{/molecule}}$$
$$= 2.05 \times 10^{-5} \text{ kg/ms}$$ *Ans.*
$$= 2.05 \times 10^{-5} \text{ Ns/m}^2$$ *Ans.*

Example 22.5

Calculate the thermal conductivity of oxygen at 1 *atm,* 300 *K.*

Solution For oxygen, a diatomic gas, the degree of freedom $f = 5$.

$$\bar{v} = [8\, KT/\pi m]^{1/2} = 445 \text{ m/s}$$
$$\sigma = \pi d^2 = 3.84 \times 10^{-19} \text{m}^2$$
$$k = \frac{1}{6}\frac{\bar{v} f K}{\sigma}$$
$$= \frac{1}{6}\frac{445 \text{ m/s} \times 5 \times 1.38 \times 10^{-23} \text{ J/molecule K}}{3.84 \times 10^{-19} \text{ m}^2\text{/molecule}}$$
$$= 0.0133 \text{ W/mK}$$ *Ans.*

If the gas has Maxwellian velocity distribution,

$$k = \frac{1}{3}\frac{f\,K}{\sigma}[KT/\pi m]^{1/2}$$

$$= \frac{1}{3}\frac{5 \times 1.38 \times 10^{-23} \text{ J/molecule K}}{3.84 \times 10^{-19} \text{m}^2\text{/molecule}} \times \left[\frac{1.38 \times 10^{-23} (\text{J/molecule K}) \times 300}{\pi \times 5.31 \times 10^{-26} \text{ kg/molecule}}\right]$$

$$= \frac{5 \times 1.38 \times 10^{-23}}{3 \times 3.84 \times 10^{-19}}\frac{J}{m^2\text{K}}\left[\frac{1.38 \times 10^{-23} \times 300}{\pi \times 5.31 \times 10^{-26}}\right]^{1/2} \text{m/s}$$

$= 0.0095$ W/mK *Ans.*

Example 22.6

Determine the pressure in a cathode-ray tube such that 90% of the electrons leaving the cathode ray reach the anode 20 cm away without making a collision. The diameter of an ion is 3.6×10^{-10} m and the electron temperature is 2000 K. Use the electronic mean free path $\lambda_e = 4/\sigma n$, where σ is the cross-section of the ion.

Solution The several equation is $N = N_0 e^{-x/\lambda}$

where $N = 0.9\,N_0$ and $x = 0.2$ m

$$0.9 = e^{-x/\lambda}$$

$$x/\lambda = 0.1053$$

$$\lambda = 0.2/0.1053 = 1.9 \text{ m}$$

$$\sigma = 4\pi r^2 = 4\pi \times (1.8 \times 10^{-10})^2 = 40.715 \times 10^{-20} \text{ m}^2$$

$$\lambda_e = \frac{4}{\sigma n} = \frac{4}{40.715 \times 10^{-20} \times n} = 1.9 \text{ m}$$

$$n = \frac{4}{40.715 \times 10^{-20} \times 1.9} = 5.17 \times 10^{18} \text{ molecules/m}^3$$

Pressure in the cathode ray tube

$$P = nKT \quad = 5.17 \times 10^{18} \times 1.38 \times 10^{-23} \times 2000 \text{ N/m}^2$$

$$= 14.27 \times 10^{-2} = 0.1427 \text{ Pa}$$ *Ans.*

Example 22.7

Oxygen gas is contained in a one-litre flask at atmospheric pressure and 300 K. (a) How many collisions per second are made by one molecule with the other molecules? (b) How many molecules strike one sq. cm of the flask per second? (c) How many molecules are there in the flask? Take radius of oxygen molecule as 1.8×10^{-10} m.

Solution (a) Number of molecules in the flask at

$$1 \text{ atm, } 300 \text{ K} = N/V = n = p/KT$$

$$= \frac{101.325 \times 1000 \text{ N/m}^2}{1.38 \times 10^{-23} \text{ J/molecule - K} \times 300 \text{ K}}$$

$= 2.45 \times 10^{25}$ molecules/m^3

$= 2.45 \times 10^{22}$ molecules/litre

$\sigma = 4\pi r^2 = 4\pi \times (1.8 \times 10^{-10})^2 = 40.71 \times 10^{-20}$ m^2

Collision frequency, $z = \sigma n \bar{v}$,

where $\bar{v} = [8KT/\pi m]^{1/2}$

Mass of an oxygen molecule, $m = 5.31 \times 10^{-26}$ kg/molecule

$$\bar{v} = \left[\frac{8 \times 1.38 \times 10^{-23} \times 300}{\pi \times 5.31 \times 10^{-26}}\right]^{1/2} = 445.58 \text{ m/s}$$

$z = 40.71 \times 10^{-20} \times 2.45 \times 10^{25} \times 445.58$

$= 4.44 \times 10^9$ collisions/s *Ans.* (a)

(b) Number of collisions $= \frac{1}{4} n \bar{v}$

$= \frac{1}{4} \times 2.45 \times 10^{25} \times 445.58$

$= 272.92 \times 10^{25}$ collisions/m^{2}s

$= 2.73 \times 10^{23}$ collisions/cm^{2}s *Ans.* (b)

(c) Number of molecules in the flask

$= 2.45 \times 10^{22}$ *Ans.* (c)

Example 22.8

A group of oxygen molecules start their free paths at the same instant. The pressure is such that the mean free path is 2 cm. After how long a time will half of the group still remain, i.e., half will not as yet have made a collision? Assume that all particles have a speed equal to the average speed. The temperature is 300 K.

Solution The survival equation in $N = N_0 e^{-x/\lambda}$

The group is reduced to half after travelling a distance x, i.e., half of the group has a free path longer than x. The mean free path, λ, is 2 cm.

$N/N_0 = 0.5 = e^{-x/\lambda}$

$x/\lambda = \ln 2 = 0.693$

$x = 1.386$ cm

For oxygen at 300K, $\bar{v} = 445.58$ m/s $= 4.456 \times 10^4$ cm/s

Time, $t = 1.386/(4.456 \times 10^4) = 3.11 \times 10^{-5}$ s *Ans.*

Example 22.9

To what pressure, in mm of Hg, must a cathode ray tube be evacuated in order that 90% of the electrons leaving the cathode shall reach the anode, 20 cm away, without making a collision? Take for ion $\sigma = 4.07 \times 10^{-19}$ m^2 and $T = 2000$ K.

Solution The survival equation is $N = N_0 e^{-x/\lambda}$

Here, $N = 0.9\, N_0$, $x = 0.2$ m

$0.9 = e^{-x/\lambda}$

$$e^{x/\lambda} = 1.111$$
$$x/\lambda = \ln 1.111$$
$$= 0.2/0.105 \cong 2 \text{ m}$$

Electronic mean free path $\lambda_e = \dfrac{1}{\sigma n} = \dfrac{1}{4.07 \times 10^{-19} \times n} = 2 \text{ m}$

$$n = 1.23 \times 10^{18} \text{ molecules/m}^3$$

Now, pressure $p = nKT$

$$= 1.23 \times 10^{18} \frac{\text{molecules}}{\text{m}^3} \times 1.38 \times 10^{-23} \frac{J}{\text{molecule K}} \times 2000 \text{ K}$$

$$= 3.395 \times 10^{-2} \text{ N/m}^2$$ *Ans.*

Example 22.10

A tube 2 m long and 10^{-4} m^2 in cross-section contain CO_2 at atmospheric pressure and 0°C. The carbon atoms in one-half of the CO_2 molecules are radioactive isotope C^{14}. At time $t = 0$, all the molecules at the extreme left end of the tube contain radioactive carbon, and the number of such molecules per unit volume decreases uniformly to zero at the other end of the tube. (a) What is the initial concentration gradient of radioactive molecules? (b) Initially, how many radioactive molecules per sec cross a cross-section at the mid-point of the tube from left to right? (c) How many cross from right to left? What is the initial net rate of diffusion of radioactive molecules across the cross-section? Take $\sigma = \pi r^2 = 4 \times 10^{-19}$ m^2.

Solution (a) Number of molecules/m³ at 1 atm, 273 K

$$n = p/KT = \frac{101.325 \times 1000}{1.38 \times 10^{-23} \times 273} = 2.69 \times 10^{25} \text{ molecules/m}^3$$

Concentration gradient, $dn/dx = (-2.69 \times 10^{25})/2$

$$= -1.345 \times 10^{25} \text{ molecules/m}^4$$ *Ans.*

(b)
$$\bar{v} = [2.55\, KT/m]^{1/2}$$
$$= \left[\frac{2.55 \times 1.38 \times 10^{-23} \times 273 \times 6.023 \times 10^{26}}{46}\right]^{1/2}$$
$$\bar{v} = 355 \text{ m/s}$$
$$\lambda = \frac{1}{\sigma n} = \frac{1}{2.69 \times 10^{25} \times 4 \times 10^{-19}} = 9.3 \times 10^{-6} \text{ m}$$

Number of molecules crossing from left to right per unit area per unit time:

$$\underset{\rightarrow}{\Gamma} = \frac{1}{4} z n_0 \lambda - \frac{1}{6} z \lambda_2 \frac{dn}{dx}$$
$$= \frac{1}{4} \bar{v} n_0 - \frac{1}{6} \bar{v} \lambda \frac{dn}{dx}$$
$$= \frac{1}{4} \times 355 \times 2.69 \times 10^{25} - \frac{1}{6} \times 355 \times 9.3 \times 10^{-6} \times (-1.345 \times 10^{25})$$
$$= 2.39 \times 10^{27} + 7.4 \times 10^{21} \text{ molecules/m}^2\text{s}$$ *Ans.*

(c) Number of molecules crossing from right to left per unit area unit time:

$$\Gamma = \frac{1}{4} v n_0 + \frac{1}{6} z\lambda \frac{dn}{dx}$$

$$= 2.39 \times 10^{27} - 7.4 \times 10^{21} \text{ molecules/m}^2\text{s} \qquad \textit{Ans.}$$

Net rate of diffusion:

$$= 7.4 \times 10^{21} \times 2 = 14.8 \times 10^{2} \text{ molecules/m}^2\text{s}$$

$$= \frac{14.8 \times 10^{21} \times 46}{6.023 \times 10^{26}} \frac{\text{molecules}}{\text{m}^2\text{s}} \times \frac{\text{kg}}{\text{kgmol}} \times \frac{\text{kgmol}}{\text{molecules}}$$

$$= 113 \times 10^{-5} = 11.3 \times 10^{-4} \text{ kg/m}^2\text{s}$$

$$= 1.13 \text{ g/m}^2\text{s} \qquad \textit{Ans.}$$

Review Questions

22.1 Define mean free path, collision cross-section and collision frequency.

22.2 Show that $\lambda = 1/\sigma n$. What is electronic mean path? Why is it is equal to $4/\sigma n$?

22.3 What is collision probability? Show that it is reciprocal of the mean free path.

22.4 Derive the survival equation: $N = N_0 e^{-x/\lambda}$ and explain its significance.

22.5 Show that 37% of the molecules in a gas have free paths longer than λ.

22.6 Explain graphically the distribution of free paths of gas molecules.

22.7 What are transport properties? What do they signify?

22.8 Show that the number of molecules crossing a plane in a gas per unit area and per unit time is equal to $\frac{1}{4} n\bar{v}$.

22.9 Show that the average distance from a plane in a gas where the molecules made their last collisions before crossing that plane is equal to $\frac{2}{3}\lambda$.

22.10 Show that the coefficient of viscosity of a gas is equal to $\frac{1}{3} nm\bar{v}\lambda$. With a Maxwellian velocity distribution of gas molecules, show that

$$\mu = \frac{2}{3\sigma}\left[\frac{m KT}{\pi}\right]^{1/2}.$$

and hence, assert that the viscosity of a gas depends only on the temperature, and is independent of pressure or density of the gas.

22.11 Show that the thermal conductivity of a gas is given by $k = \frac{1}{6}\frac{\bar{v} f K}{\sigma}$. With a Maxwellian velocity distribution, show that:

$$k = \frac{1}{3}\frac{f K}{\sigma}\left[\frac{KT}{\pi m}\right]^{1/2}$$

Hence, assert that the thermal conductivity of a gas is independent of pressure or density, and depends only on the temperature.

22.12 Show that for a monatomic gas,

$$k = \frac{1}{\pi^{3/2} d^2}\left[\frac{K^3 T}{m}\right]^{1/2}$$

22.13 Show that the theoretical value of Prandtl number of a gas is equal to the specific heat ratio of the gas.

22.14 What do you understand by a self-diffusion process? What is the coefficient of diffusion?

22.15 Derive the expression for the coefficient of diffusion in a gas from molecular theory as given by:

$$D = \frac{1}{3}\bar{v}\lambda$$

With Maxwellian velocity distribution, show that

$$D = \frac{2}{3\pi^{1/2} d^2 n_t}\left[\frac{K T}{m}\right]^{1/2}$$

Where n_t is the total number of molecules per unit volume.

22.16 Define Schmidt number. What is its physical significance? Show that for a gas the Schmidt number is unity.

22.17 Why do all collisions of gas molecules not cause ionization?

22.18 Explain why at high temperatures a gas can be highly conductive of electricity.

22.19 Show that the ratio of velocities of an ion and a free electron is given by:

$$v_i/v_e = 1/1840M$$

where M is the atomic weight of the ion.

22.20 Explain the distribution of collision times of electrons.

22.21 What do you mean by electron drift velocity? Show that it is given by:

$$v_{e_d} = \frac{q_e E\tau_e}{m_e} = \frac{4q_e E}{\sigma p}\left[\frac{\pi KT}{8m_e}\right]^{1/2}$$

22.22 Show that the electrical conductivity of a gas is given by:

$$\sigma_e = \frac{q_e^2 n_e}{\sigma n}\left[\frac{2\pi}{m_e KT}\right]^{1/2}$$

22.23 Define electron mobility. Show that it is given by:

$$\mu_e = \frac{q_e}{\sigma n}\left[\frac{2\pi}{m_e KT}\right]^{1/2}$$

Problems

22.1 Calculate the collision frequency of a nitrogen molecule (a) at 300 K and 1 atm pressure, (b) at 300 K and 1 micron Hg abs. pressure. The radius of nitrogen molecule is 1.88×10^{-10}m.

Ans. (a) 7.35×10^9 collisions/s
(b) 9.63×10^3 collisions/s

22.2 Calculate the collision rate of a molecule in a Maxwellian gas. *Ans.* $\sqrt{2}\,\sigma n[8\text{ KT}/\pi m]^{1/2}$

22.3 The mean free path of a certain gas in 12 cm. If there are 10,000 free paths, how many are longer than (a) 5 cm, (b) 15 cm, (c) 50 cm? (d) How many are longer than 6 cm, but shorter than 12 cm? (e) How many are between 11.5 cm and 12.5 cm in length? (f) How many are between 11.9 and 12.1 cm in length? (g) How many have free paths exactly equal to 12cm?

22.4 The mean free path of the molecules of a certain gas at 20°C is 3×10^{-5} m. (a) If the radius of the molecule is 3×10^{-10} m, find the pressure of the gas. (b) Calculate the number of collisions made by a molecule per metre of path.

22.5 The mean free path of the molecules of a certain gas at 298 K is 2.63×10^{-5} mm, the radius of each molecule is 2.56×10^{-10} m. Compute the number of collisions made by a typical particle in moving a distance of 1 m, and also the pressure of the gas.

22.6 Determine the pressure in a cathode ray tube such that 95 per cent of the electrons leaving the cathode ray reach the anode 25 cm away without making a collision. The diameter of an ion is 3.6×10^{-10} m and the electron temperature is 2000 K. Use the electronic mean free path $\lambda_e = 4/\sigma n$, where σ is the cross-section of the ion.

22.7 A beam of electrons is projected from an electron gun into a gas at a pressure p, and the number remaining in the beam at a distance x from the gun is determined by allowing the beam to strike a collecting plate and measuring the current to the plate. The electron current emitted by the gun is 100 μa, and the current to the plate when x = 10 cm and p = 1 mmHg is 37 μa. Determine (a) the electron mean free path, and (b) the current at 500 μ Hg pressure. *Ans.* (a) 10 cm, (b) 60.7 μa

22.8 A singly charged oxygen ion starts a free path in a direction at right angles to an electric field of intensity 100 volts/cm. The pressure is one atmosphere and the temperature 300 K. Calculate (a) the distance moved in the direction of the field in a time equal to that required to traverse one mean free path, (b) the ratio of the mean free path to this distance, (c) the average velocity in the direction of the field, (d) the ratio of the thermal velocity to this velocity, and (e) the ratio of the energy of thermal agitation to the energy gained from the field in one mean free path.

Ans. (a) 3.87×10^{-10} m, (c) 340 m/s, (e)10^4

22.9 A spherical satellite d metre in diameter moves through the earth's atmosphere with a speed of $\bar{v}$ m/s at an altitude where the number density is n molecules/m^3. How many molecules strike the satellite in 1 second? Derive an expression for the drag experienced by the satellite, assuming that all molecules which strike the sphere adhere to it.

Appendix A

TABLE A.1 STEAM TABLES*

Table A.1.1
Saturated Steam: Temperature Table

Temp. °C T	Pressure kPa, MPa P	Specific Volume, m^3/kg Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
0.01	0.6113	0.001000	206.132	0.00	2375.3	2375.3	0.00	2501.3	2501.3	0.0000	9.1562	9.1562
5	0.8721	0.001000	147.118	20.97	2361.3	2382.2	20.98	2489.6	2510.5	0.0761	8.9496	9.0257
10	1.2276	0.001000	106.377	41.99	2347.2	2389.2	41.99	2477.7	2519.7	0.1510	8.7498	8.9007
15	1.7051	0.001001	77.925	62.98	2333.1	2396.0	62.98	2465.9	2528.9	0.2245	8.5569	8.7813
20	2.3385	0.001002	57.790	83.94	2319.0	2402.9	83.94	2454.1	2538.1	0.2966	8.3706	8.6671
25	3.1691	0.001003	43.359	104.86	2304.9	2409.8	104.87	2442.3	2547.2	0.3673	8.1905	8.5579
30	4.2461	0.001004	32.893	125.77	2290.8	2416.6	125.77	2430.5	2556.2	0.4369	8.0164	8.4533
35	5.6280	0.001006	25.216	146.65	2276.7	2423.4	146.66	2418.6	2565.3	0.5052	7.8478	8.3530
40	7.3837	0.001008	19.523	167.53	2262.6	2430.1	167.54	2406.7	2574.3	0.5724	7.6845	8.2569
45	9.5934	0.001010	15.258	188.41	2248.4	2436.8	188.42	2394.8	2583.2	0.6386	7.5261	8.1647

* Adapted from Joseph H. Keenan, Frederick G. Keyes, Philip G. Hill, and Joan G. Moore, *Steam Tables*, John Wiley and Sons, New York, 1969.

(*Continued*)

Table A.1.1
(Continued)

Temp. °C T	Pressure kPa, MPa P	Specific Volume, m³/kg Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
50	12.350	0.001012	12.032	209.30	2234.2	2443.5	209.31	2382.7	2592.1	0.7037	7.3725	8.0762
55	15.758	0.001015	9.568	230.19	2219.9	2450.1	230.20	2370.7	2600.9	0.7679	7.2234	7.9912
60	19.941	0.001017	7.671	251.09	2205.5	2456.6	251.11	2358.5	2609.6	0.8311	7.0784	7.9095
65	25.033	0.001020	6.197	272.00	2191.1	2463.1	272.03	2346.2	2618.2	0.8934	6.9375	7.8309
70	31.188	0.001023	5.042	292.93	2176.6	2469.5	292.96	2333.8	2626.8	0.9548	6.8004	7.7552
75	38.578	0.001026	4.131	313.87	2162.0	2475.9	313.91	2321.4	2635.3	1.0154	6.6670	7.6824
80	47.390	0.001029	3.407	334.84	2147.4	2482.2	334.88	2308.8	2643.7	1.0752	6.5369	7.6121
85	57.834	0.001032	2.828	355.82	2132.6	2488.4	355.88	2296.0	2651.9	1.1342	6.4102	7.5444
90	70.139	0.001036	2.361	376.82	2117.7	2494.5	376.90	2283.2	2660.1	1.1924	6.2866	7.4790
95	84.554	0.001040	1.982	397.86	2102.7	2500.6	397.94	2270.2	2668.1	1.2500	6.1659	7.4158
100	0.10135	0.001044	1.6729	418.91	2087.6	2506.5	419.02	2257.0	2676.0	1.3068	6.0480	7.3548
105	0.12082	0.001047	1.4194	440.00	2072.3	2512.3	440.13	2243.7	2683.8	1.3629	5.9328	7.2958
110	0.14328	0.001052	1.2102	461.12	2057.0	2518.1	461.27	2230.2	2691.5	1.4184	5.8202	7.2386
115	0.16906	0.001056	1.0366	482.28	2041.4	2523.7	482.46	2216.5	2699.0	1.4733	5.7100	7.1832
120	0.19853	0.001060	0.8919	503.48	2025.8	2529.2	503.69	2202.6	2706.3	1.5275	5.6020	7.1295
125	0.2321	0.001065	0.77059	524.72	2009.9	2534.6	524.96	2188.5	2713.5	1.5812	5.4962	7.0774
130	0.2701	0.001070	0.66850	546.00	1993.9	2539.9	546.29	2174.2	2720.5	1.6343	5.3925	7.0269
135	0.3130	0.001075	0.58217	567.34	1977.7	2545.0	567.67	2159.6	2727.3	1.6869	5.2907	6.9777
140	0.3613	0.001080	0.50885	588.72	1961.3	2550.0	589.11	2144.8	2733.9	1.7390	5.1908	6.9298
145	0.4154	0.001085	0.44632	610.16	1944.7	2554.9	610.61	2129.6	2740.3	1.7906	5.0926	6.8832
150	0.4759	0.001090	0.39278	631.66	1927.9	2559.5	632.18	2114.3	2746.4	1.8417	4.9960	6.8378

155	0.5431	0.001096	0.34676	653.23	1910.8	2564.0	653.82	2098.6	2752.4	1.8924	4.9010	6.7934
160	0.6178	0.001102	0.30706	674.85	1893.5	2568.4	675.53	2082.6	2758.1	1.9426	4.8075	6.7501
165	0.7005	0.001108	0.27269	696.55	1876.0	2572.5	697.32	2066.2	2763.5	1.9924	4.7153	6.7078
170	0.7917	0.001114	0.24283	718.31	1858.1	2576.5	719.20	2049.5	2768.7	2.0418	4.6244	6.6663
175	0.8920	0.001121	0.21680	740.16	1840.0	2580.2	741.16	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0022	0.001127	0.19405	762.08	1821.6	2583.7	763.21	2015.0	2778.2	2.1395	4.4461	6.5857
185	1.1227	0.001134	0.17409	784.08	1802.9	2587.0	785.36	1997.1	2782.4	2.1878	4.3586	6.5464
190	1.2544	0.001141	0.15654	806.17	1783.8	2590.0	807.61	1978.8	2786.4	2.2358	4.2720	6.5078
195	1.3978	0.001149	0.14105	828.36	1764.4	2592.8	829.96	1960.0	2790.0	2.2835	4.1863	6.4697
200	1.5538	0.001156	0.12736	850.64	1744.7	2595.3	852.43	1940.7	2793.2	2.3308	4.1014	6.4322
205	1.7230	0.001164	0.11521	873.02	1724.5	2597.5	875.03	1921.0	2796.0	2.3779	4.0172	6.3951
210	1.9063	0.001173	0.10441	895.51	1703.9	2599.4	897.75	1900.7	2798.5	2.4247	3.9337	6.3584
215	2.1042	0.001181	0.09479	918.12	1682.9	2601.1	920.61	1879.9	2800.5	2.4713	3.8507	6.3221
220	2.3178	0.001190	0.08619	940.85	1661.5	2602.3	943.61	1858.5	2802.1	2.5177	3.7683	6.2860
225	2.5477	0.001199	0.07849	963.72	1639.6	2603.3	966.77	1836.5	2803.3	2.5639	3.6863	6.2502
230	2.7949	0.001209	0.07158	986.72	1617.2	2603.9	990.10	1813.8	2803.9	2.6099	3.6047	6.2146
235	3.0601	0.001219	0.06536	1009.88	1594.2	2604.1	1013.61	1790.5	2804.1	2.6557	3.5233	6.1791
240	3.3442	0.001229	0.05976	1033.19	1570.8	2603.9	1037.31	1766.5	2803.8	2.7015	3.4422	6.1436
245	3.6482	0.001240	0.05470	1056.69	1546.7	2603.4	1061.21	1741.7	2802.9	2.7471	3.3612	6.1083
250	3.9730	0.001251	0.05013	1080.37	1522.0	2602.4	1085.34	1716.2	2801.5	2.7927	3.2802	6.0729
255	4.3195	0.001263	0.04598	1104.26	1496.7	2600.9	1109.72	1689.8	2799.5	2.8382	3.1992	6.0374
260	4.6886	0.001276	0.04220	1128.37	1470.6	2599.0	1134.35	1662.5	2796.9	2.8837	3.1181	6.0018
265	5.0813	0.001289	0.03877	1152.72	1443.9	2596.6	1159.27	1634.3	2793.6	2.9293	3.0368	5.9661
270	5.4987	0.001302	0.03564	1177.33	1416.3	2593.7	1184.49	1605.2	2789.7	2.9750	2.9551	5.9301
275	5.9418	0.001317	0.03279	1202.23	1387.9	2590.2	1210.05	1574.9	2785.0	3.0208	2.8730	5.8937

(Continued)

Table A.1.1
(*Continued*)

Temp. °C T	Pressure kPa, MPa P	Specific Volume, m³/kg: Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg: Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg: Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K: Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
280	6.4117	0.001332	0.03017	1227.43	1358.7	2586.1	1235.97	1543.6	2779.5	3.0667	2.7903	5.8570
285	6.9094	0.001348	0.02777	1252.98	1328.4	2581.4	1262.29	1511.0	2773.3	3.1129	2.7069	5.8198
290	7.4360	0.001366	0.02557	1278.89	1297.1	2576.0	1289.04	1477.1	2766.1	3.1593	2.6227	5.7821
295	7.9928	0.001384	0.02354	1305.21	1264.7	2569.9	1316.27	1441.8	2758.0	3.2061	2.5375	5.7436
300	8.5810	0.001404	0.02167	1331.97	1231.0	2563.0	1344.01	1404.9	2748.9	3.2533	2.4511	5.7044
305	9.2018	0.001425	0.01995	1359.22	1195.9	2555.2	1372.33	1366.4	2738.7	3.3009	2.3633	5.6642
310	9.8566	0.001447	0.01835	1387.03	1159.4	2546.4	1401.29	1326.0	2727.3	3.3492	2.2737	5.6229
315	10.547	0.001472	0.01687	1415.44	1121.1	2536.6	1430.97	1283.5	2714.4	3.3981	2.1821	5.5803
320	11.274	0.001499	0.01549	1444.55	1080.9	2525.5	1461.45	1238.6	2700.1	3.4479	2.0882	5.5361
330	12.845	0.001561	0.012996	1505.24	993.7	2498.9	1525.29	1140.6	2665.8	3.5506	1.8909	5.4416
340	14.586	0.001638	0.010797	1570.26	894.3	2464.5	1594.15	1027.9	2622.0	3.6593	1.6763	5.3356
350	16.514	0.001740	0.008813	1641.81	776.6	2418.4	1670.54	893.4	2563.9	3.7776	1.4336	5.2111
360	18.651	0.001892	0.006945	1725.19	626.3	2351.5	1760.48	720.5	2481.0	3.9146	1.1379	5.0525
370	21.028	0.002213	0.004926	1843.84	384.7	2228.5	1890.37	441.8	2332.1	4.1104	0.6868	4.7972
374.14	22.089	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

Table A.1.2
Saturated Water: Pressure Table

Pressure kPa P	Temp. °C T	Specific Volume, m³/kg Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Evap. s_{fg}	Sat Vapour s_g
0.6113	0.01	0.001000	206.132	0	2375.3	2375.3	0.00	2501.3	2501.3	0	9.1562	9.1562
1.0	6.98	0.001000	129.208	29.29	2355.7	2385.0	29.29	2484.9	2514.2	0.1059	8.8697	8.9756
1.5	13.03	0.001001	87.980	54.70	2338.6	2393.3	54.70	2470.6	2525.3	0.1956	8.6322	8.8278
2.0	17.50	0.001001	67.004	73.47	2326.0	2399.5	73.47	2460.0	2533.5	0.2607	8.4629	8.7236
2.5	21.08	0.001002	54.254	88.47	2315.9	2404.4	88.47	2451.6	2540.0	0.3120	8.3311	8.6431
3.0	24.08	0.001003	45.665	101.03	2307.5	2408.5	101.03	2444.5	2545.5	0.3545	8.2231	8.5775
4.0	28.96	0.001004	34.800	121.44	2293.7	2415.2	121.44	2432.9	2554.4	0.4226	8.0520	8.4746
5.0	32.88	0.001005	28.193	137.79	2282.7	2420.5	137.79	2423.7	2561.4	0.4763	7.9187	8.3950
7.5	40.29	0.001008	19.238	168.76	2261.7	2430.5	168.77	2406.0	2574.8	0.5763	7.6751	8.2514
10.0	45.81	0.001010	14.674	191.79	2246.1	2437.9	191.81	2392.8	2584.6	0.6492	7.5010	8.1501
15.0	53.97	0.001014	10.022	225.90	2222.8	2448.7	225.91	2373.1	2599.1	0.7548	7.2536	8.0084
20.0	60.06	0.001017	7.649	251.35	2205.4	2456.7	251.38	2358.3	2609.7	0.8319	7.0766	7.9085
25.0	64.97	0.001020	6.204	271.88	2191.2	2463.1	271.90	2346.3	2618.2	0.8930	6.9383	7.8313
30.0	69.10	0.001022	5.229	289.18	2179.2	2468.4	289.21	2336.1	2626.3	0.9439	6.8247	7.7686
40.0	75.87	0.001026	3.993	317.51	2159.5	2477.0	317.55	2319.2	2636.7	1.0258	6.6441	7.6700
50.0	81.33	0.001030	3.240	340.42	2143.4	2483.8	340.47	2305.4	2645.9	1.0910	6.5029	7.5939

(Continued)

Table A.1.2
(*Continued*)

Pressure kPa P	Temp. °C T	Specific Volume, m³/kg: Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg: Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg: Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K: Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
75.0	91.77	0.001037	2.217	384.29	2112.4	2496.7	384.36	2278.6	2663.0	1.2129	6.2434	7.4563
MPa												
0.100	99.62	0.001043	1.6940	417.33	2088.7	2506.1	417.44	2258.0	2675.5	1.3025	6.0568	7.3593
0.125	105.99	0.001048	1.3749	444.16	2069.3	2513.5	444.30	2241.1	2685.3	1.3739	5.9104	7.2843
0.150	111.37	0.001053	1.1593	466.92	2052.7	2519.6	467.08	2226.5	2693.5	1.4335	5.7897	7.2232
0.175	116.06	0.001057	1.0036	486.78	2038.1	2524.9	486.97	2213.6	2700.5	1.4848	5.6868	7.1717
0.200	120.23	0.001061	0.8857	504.47	2025.0	2529.5	504.68	2202.0	2706.6	1.5300	5.5970	7.1271
0.225	124.00	0.001064	0.7933	520.45	2013.1	2533.6	520.69	2191.3	2712.0	1.5705	5.5173	7.0878
0.250	127.43	0.001067	0.7187	535.08	2002.1	2537.2	535.34	2181.5	2716.9	1.6072	5.4455	7.0526
0.275	130.60	0.001070	0.6573	548.57	1992.0	2540.5	548.87	2172.4	2721.3	1.6407	5.3801	7.0208
0.300	133.55	0.001073	0.6058	561.13	1982.4	2543.6	561.45	2163.9	2725.3	1.6717	5.3201	6.9918
0.325	136.30	0.001076	0.5620	572.88	1973.5	2546.3	573.23	2155.8	2729.0	1.7005	5.2646	6.9651
0.350	138.88	0.001079	0.5243	583.93	1965.0	2548.9	584.31	2148.1	2732.4	1.7274	5.2130	6.9404
0.375	141.32	0.001081	0.4914	594.38	1956.9	2551.3	594.79	2140.8	2735.6	1.7527	5.1647	6.9174
0.40	143.63	0.001084	0.4625	604.29	1949.3	2553.6	604.73	2133.8	2738.5	1.7766	5.1193	6.8958
0.45	147.93	0.001088	0.4140	622.75	1934.9	2557.6	623.24	2120.7	2743.9	1.8206	5.0359	6.8565
0.50	151.86	0.001093	0.3749	639.66	1921.6	2561.2	640.21	2108.5	2748.7	1.8606	4.9606	6.8212
0.55	155.48	0.001097	0.3427	655.30	1909.2	2564.5	655.91	2097.0	2752.9	1.8972	4.8920	6.7892
0.60	158.85	0.001101	0.3157	669.88	1897.5	2567.4	670.54	2086.3	2756.8	1.9311	4.8289	6.7600
0.65	162.01	0.001104	0.2927	683.55	1886.5	2570.1	684.26	2076.0	2760.3	1.9627	4.7704	6.7330

0.70	164.97	0.001108	0.2729	696.43	1876.1	2572.5	697.20	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.77	0.001111	0.2556	708.62	1866.1	2574.7	709.45	2057.0	2766.4	2.0199	4.6647	6.6846
0.80	170.43	0.001115	0.2404	720.20	1856.6	2576.8	721.10	2048.0	2769.1	2.0461	4.6166	6.6627
0.85	172.96	0.001118	0.2270	731.25	1847.4	2578.7	732.20	2039.4	2771.6	2.0709	4.5711	6.6421
0.90	175.38	0.001121	0.2150	741.81	1838.7	2580.5	742.82	2031.1	2773.9	2.0946	4.5280	6.6225
0.95	177.69	0.001124	0.2042	751.94	1830.2	2582.1	753.00	2023.1	2776.1	2.1171	4.4869	6.6040
1.00	179.91	0.001127	0.19444	761.67	1822.0	2583.6	762.79	2015.3	2778.1	2.1386	4.4478	6.5864
1.10	184.09	0.001133	0.17753	780.08	1806.3	2586.4	781.32	2000.4	2781.7	2.1791	4.3744	6.5535
1.20	187.99	0.001139	0.16333	797.27	1791.6	2588.8	798.64	1986.2	2784.8	2.2165	4.3067	6.5233
1.30	191.64	0.001144	0.15125	813.42	1777.5	2590.9	814.91	1972.7	2787.6	2.2514	4.2438	6.4953
1.40	195.07	0.001149	0.14084	828.68	1764.1	2592.8	830.29	1959.7	2790.0	2.2842	4.1850	6.4692
1.50	198.32	0.001154	0.13177	843.14	1751.3	2594.5	844.87	1947.3	2792.1	2.3150	4.1298	6.4448
1.75	205.76	0.001166	0.11349	876.44	1721.4	2597.8	878.48	1918.0	2796.4	2.3851	4.0044	6.3895
2.00	212.42	0.001177	0.09963	906.42	1693.8	2600.3	908.77	1890.7	2799.5	2.4473	3.8935	6.3408
2.25	218.45	0.001187	0.08875	933.81	1668.2	2602.0	936.48	1865.2	2801.7	2.5034	3.7938	6.2971
2.50	223.99	0.001197	0.07998	959.09	1644.0	2603.1	962.09	1841.0	2803.1	2.5546	3.7028	6.2574
2.75	229.12	0.001207	0.07275	982.65	1621.2	2603.8	985.97	1817.9	2803.9	2.6018	3.6190	6.2208
3.00	233.90	0.001216	0.06668	1004.76	1599.3	2604.1	1008.41	1795.7	2804.1	2.6456	3.5412	6.1869
3.25	238.38	0.001226	0.06152	1025.62	1578.4	2604.0	1029.60	1774.4	2804.0	2.6866	3.4685	6.1551
3.50	242.60	0.001235	0.05707	1045.41	1558.3	2603.7	1049.73	1753.7	2803.4	2.7252	3.4000	6.1252
4.0	250.40	0.001252	0.049778	1082.28	1520.0	2602.3	1087.29	1714.1	2801.4	2.7963	3.2737	6.0700
5.0	263.99	0.001286	0.039441	1147.78	1449.3	2597.1	1154.21	1640.1	2794.3	2.9201	3.0532	5.9733

(Continued)

Table A1.2
(Continued)

Pressure kPa P	Temp. °C T	Specific Volume, m³/kg Sat. Liquid v_f	Specific Volume, m³/kg Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Internal Energy, kJ/kg Evap. u_{fg}	Internal Energy, kJ/kg Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Enthalpy, kJ/kg Evap. h_{fg}	Enthalpy, kJ/kg Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Entropy, kJ/kg K Evap. s_{fg}	Entropy, kJ/kg K Sat. Vapour s_g
6.0	275.64	0.001319	0.032440	1205.41	1384.3	2589.7	1213.32	1571.0	2784.3	3.0266	2.8625	5.8891
7.0	285.88	0.001351	0.027370	1257.51	1323.0	2580.5	1266.97	1505.1	2772.1	3.1210	2.6922	5.8132
8.0	295.06	0.001384	0.023518	1305.54	1264.3	2569.8	1316.61	1441.3	2757.9	3.2067	2.5365	5.7431
9.0	303.40	0.001418	0.020484	1350.47	1207.3	2557.8	1363.23	1378.9	2742.1	3.2857	2.3915	5.6771
10.0	311.06	0.001452	0.018026	1393.00	1151.4	2544.4	1407.53	1317.1	2724.7	3.3595	2.2545	5.6140
11.0	318.15	0.001489	0.015987	1433.68	1096.1	2529.7	1450.05	1255.5	2705.6	3.4294	2.1233	5.5527
12.0	324.75	0.001527	0.014263	1472.92	1040.8	2513.7	1491.24	1193.6	2684.8	3.4961	1.9962	5.4923
13.0	330.93	0.001567	0.012780	1511.09	985.0	2496.1	1531.46	1130.8	2662.2	3.5604	1.8718	5.4323
14.0	336.75	0.001611	0.011485	1548.53	928.2	2476.8	1571.08	1066.5	2637.5	3.6231	1.7485	5.3716
15.0	342.24	0.001658	0.010338	1585.58	869.8	2455.4	1610.45	1000.0	2610.5	3.6847	1.6250	5.3097
16.0	347.43	0.001711	0.009306	1622.63	809.1	2431.7	1650.00	930.6	2580.6	3.4760	1.4995	5.2454
17.0	352.37	0.001770	0.008365	1660.16	744.8	2405.0	1690.25	856.9	2547.2	3.8078	1.3698	5.1776
18.0	357.06	0.001840	0.007490	1698.86	675.4	2374.3	1731.97	777.1	2509.1	3.8713	1.2330	5.1044
19.0	361.54	0.001924	0.006657	1739.87	598.2	2338.1	1776.43	688.1	2464.5	3.9387	1.0841	5.0227
20.0	365.81	0.002035	0.005834	1785.47	507.6	2293.1	1826.18	583.6	2409.7	4.0137	0.9132	4.9269
21.0	369.89	0.002206	0.004953	1841.97	388.7	2230.7	1888.30	446.4	2334.7	4.1073	0.6942	4.8015
22.0	373.80	0.002808	0.003526	1973.16	108.2	2081.4	2034.92	124.0	2159.0	4.3307	0.1917	4.5224
22.09	374.14	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

Table A.13
Superheated Vapour

T	v	u	h	s	v	u	h	s	v	u	h	s
	P = 10 kPa (45.81)				*P* = 50 kPa, (81.33)				*P* = 100 kPa (99.62)			
Sat.	14.674	2437.9	2584.6	8.1501	3.240	2483.8	2645.9	7.5939	1.6940	2506.1	2675.5	7.3593
50	14.869	2443.9	2592.6	8.1749	—	—	—	—	—	—	—	—
100	17.196	2515.5	2687.5	8.4479	3.418	2511.6	2682.5	7.6947	1.6958	2506.6	2676.2	7.3614
150	19.513	2587.9	2783.0	8.6881	3.889	2585.6	2780.1	7.9400	1.9364	2582.7	2776.4	7.6133
200	21.825	2661.3	2879.5	8.9037	4.356	2659.8	2877.6	8.1579	2.1723	2658.0	2875.3	7.8342
250	24.136	2736.0	2977.3	9.1002	4.821	2735.0	2976.0	8.3555	2.4060	2733.7	2974.3	8.0332
300	26.445	2812.1	3076.5	9.2812	5.284	2811.3	3075.5	8.5372	2.6388	2810.4	3074.3	8.2157
400	31.063	2968.9	3279.5	9.6076	6.209	2968.4	3278.9	8.8641	3.1026	2967.8	3278.1	8.5434
500	35.679	3132.3	3489.0	9.8977	7.134	3131.9	3488.6	9.1545	3.5655	3131.5	3488.1	8.8341
600	40.295	3302.5	3705.4	10.1608	8.058	3302.2	3705.1	9.4177	4.0278	3301.9	3704.7	9.0975
700	44.911	3479.6	3928.7	10.4028	8.981	3479.5	3928.5	9.6599	4.4899	3479.2	3928.2	9.3398
800	49.526	3663.8	4159.1	10.6281	9.904	3663.7	4158.9	9.8852	4.9517	3663.5	4158.7	9.5652
900	54.141	3855.0	4396.4	10.8395	10.828	3854.9	4396.3	10.0967	5.4135	3854.8	4396.1	9.7767
1000	58.757	4053.0	4640.6	11.0392	11.751	4052.9	4640.5	10.2964	5.8753	4052.8	4640.3	9.9764
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4858	6.3370	4257.3	4890.9	10.1658
1200	67.987	4467.9	5147.8	11.4090	13.597	4467.8	5147.7	10.6662	6.7986	4467.7	5147.6	10.3462
1300	72.603	4683.7	5409.7	11.5810	14.521	4683.6	5409.6	10.8382	7.2603	4683.5	5409.5	10.5182
	P = 200 kPa (120.23)				*P* = 300 kPa (133.55)				*P* = 400 kPa (143.63)			
Sat.	0.88573	2529.5	2706.6	7.1271	0.60582	2543.6	2725.3	6.9918	0.46246	2553.6	2738.5	6.8958
150	0.95964	2576.9	2768.8	7.2795	0.63388	2570.8	2761.0	7.0778	0.47084	2564.5	2752.8	6.9299
200	1.08034	2654.4	2870.5	7.5066	0.71629	2650.7	2865.5	7.3115	0.53422	2646.8	2860.5	7.1706
250	1.19880	2731.2	2971.0	7.7085	0.79636	2728.7	2967.6	7.5165	0.59512	2726.1	2964.2	7.3788

(*Continued*)

Table A.1.3
(Continued)

T	v	u	h	s	v	u	h	s	v	u	h	s
	P = 200 kPa (120.23)				P = 300 kPa (133.55)				P = 400 kPa (143.63)			
300	1.31616	2808.6	3071.8	7.8926	0.87529	2806.7	3069.3	7.7022	0.65484	2804.8	3066.7	7.5661
400	1.54930	2966.7	3276.5	8.2217	1.03151	2965.5	3275.0	8.0329	0.77262	2964.4	3273.4	7.8984
500	1.78139	3130.7	3487.0	8.5132	1.18669	3130.0	3486.0	8.3250	0.88934	3129.2	3484.9	8.1912
600	2.01297	3301.4	3704.0	8.7769	1.34136	3300.8	3703.2	8.5892	1.00555	3300.2	3702.4	8.4557
700	2.24426	3478.8	3927.7	9.0194	1.49573	3478.4	3927.1	8.8319	1.12147	3477.9	3926.5	8.6987
800	2.47539	3663.2	4158.3	9.2450	1.64994	3662.9	4157.8	9.0575	1.23722	3662.5	4157.4	8.9244
900	2.70643	3854.5	4395.8	9.4565	1.80406	3854.2	4395.4	9.2691	1.35288	3853.9	4395.1	9.1361
1000	2.93740	4052.5	4640.0	9.6563	1.95812	4052.3	4639.7	9.4689	1.46847	4052.0	4639.4	9.3360
1100	3.16834	4257.0	4890.7	9.8458	2.11214	4256.8	4890.4	9.6585	1.58404	4256.5	4890.1	9.5255
1200	3.39927	4467.5	5147.3	10.0262	2.26614	4467.2	5147.1	9.8389	1.69958	4467.0	5146.8	9.7059
1300	3.63018	4683.2	5409.3	10.1982	2.42013	4683.0	5409.0	10.0109	1.81511	4682.8	5408.8	9.8780
	P = 500 kPa (151.86)				P = 600 kPa (158.85)				P = 800 kPa (170.43)			
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
300	0.52256	2802.9	3064.2	7.4598	0.43437	2801.0	3061.6	7.3723	0.32411	2797.1	3056.4	7.2372
350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
500	0.71093	3128.4	3483.8	8.0872	0.59199	3127.6	3482.7	8.0020	0.44331	3125.9	3480.6	7.8672
600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
700	0.89691	3477.5	3926.0	8.5952	0.74720	3477.1	3925.4	8.5107	0.56007	3476.2	3924.3	8.3770
800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153
1000	1.17469	4051.8	4639.1	9.2328	0.97883	4051.5	4638.8	9.1484	0.73401	4051.0	4638.2	9.0153
1100	1.26718	4256.3	4889.9	9.4224	1.05594	4256.1	4889.6	9.3381	0.79188	4255.6	4889.1	9.2049
1200	1.35964	4466.8	5146.6	9.6028	1.13302	4466.5	5146.3	9.5185	0.84974	4466.1	5145.8	9.3854
1300	1.45210	4682.5	5408.6	9.7749	1.21009	4682.3	5408.3	9.6906	0.90758	4681.8	5407.9	9.5575

Sat.	0.19444	2583.6	2778.1	6.5864	0.16333	2588.8	2784.8	6.5233	0.14084	2592.8	2790.0	6.4692
200	0.20596	2621.9	2827.9	6.6939	0.16930	2612.7	2815.9	6.5898	0.14302	2603.1	2803.3	6.4975
250	0.23268	2709.9	2942.6	6.9246	0.19235	2704.2	2935.0	6.8293	0.16350	2698.3	2927.2	6.7467
300	0.25794	2793.2	3051.2	7.1228	0.21382	2789.2	3045.8	7.0316	0.18228	2785.2	3040.4	6.9533
350	0.28247	2875.2	3157.7	7.3010	0.23452	2872.2	3153.6	7.2120	0.20026	2869.1	3149.5	7.1359
400	0.30659	2957.3	3263.9	7.4650	0.25480	2954.9	3260.7	7.3773	0.21780	2952.5	3257.4	7.3025
500	0.35411	3124.3	3478.4	7.7621	0.29463	3122.7	3476.3	7.6758	0.25215	3121.1	3474.1	7.6026
600	0.40109	3296.8	3697.9	8.0289	0.33393	3295.6	3696.3	7.9434	0.28596	3294.4	3694.8	7.8710
700	0.44779	3475.4	3923.1	8.2731	0.37294	3474.5	3922.0	8.1881	0.31947	3473.6	3920.9	8.1160
800	0.49432	3660.5	4154.8	8.4996	0.41177	3659.8	4153.9	8.4149	0.35281	3659.1	4153.0	8.3431
900	0.54075	3852.5	4392.9	8.7118	0.45051	3851.6	4392.2	8.6272	0.38606	3851.0	4391.5	8.5555
1000	0.58712	4050.5	4637.6	8.9119	0.48919	4050.0	4637.0	8.8274	0.41924	4049.5	4636.4	8.7558
1100	0.63345	4255.1	4888.5	9.1016	0.52783	4254.6	4888.0	9.0171	0.45239	4254.1	4887.5	8.9456
1200	0.67977	4465.6	5145.4	9.2821	0.56646	4465.1	5144.9	9.1977	0.48552	4464.6	5144.4	9.1262
1300	0.72608	4681.3	5407.4	9.4542	0.60507	4680.9	5406.9	9.3698	0.51864	4680.4	5406.5	9.2983
		$P = 1.60$ MPa (201.40)				$P = 1.80$ MPa (207.15)				$P = 2.00$ MPa (212.42)		
Sat.	0.12380	2595.9	2794.0	6.4217	0.11042	2598.4	2797.1	6.3793	0.09963	2600.3	2799.5	6.3408
225	0.13287	2644.6	2857.2	6.5518	0.11673	2636.6	2846.7	6.4807	0.10377	2628.3	2835.8	6.4146
250	0.14184	2692.3	2919.2	6.6732	0.12497	2686.0	2911.0	6.6066	0.11144	2679.6	2902.5	6.5452
300	0.15862	2781.0	3034.8	6.8844	0.14021	2776.8	3029.2	6.8226	0.12547	2772.6	3023.5	6.7663
350	0.17456	2866.0	3145.4	7.0693	0.15457	2862.9	3141.2	7.0099	0.13857	2859.8	3137.0	6.9562
400	0.19005	2950.1	3254.2	7.2373	0.16847	2947.7	3250.9	7.1793	0.15120	2945.2	3247.6	7.1270
500	0.22029	3119.5	3471.9	7.5389	0.19550	3117.8	3469.7	7.4824	0.17568	3116.2	3467.6	7.4316
600	0.24998	3293.3	3693.2	7.8080	0.22199	3292.1	3691.7	7.7523	0.19960	3290.9	3690.1	7.7023
700	0.27937	3472.7	3919.7	8.0535	0.24818	3471.9	3918.6	7.9983	0.22323	3471.0	3917.5	7.9487
800	0.30859	3658.4	4152.1	8.2808	0.27420	3657.7	4151.3	8.2258	0.24668	3657.0	4150.4	8.1766

(Continued)

Table A.1.3
(*Continued*)

	P = 1.60 MPa (201.40)				*P* = 1.80 MPa (207.15)				*P* = 2.00 MPa (212.42)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
900	0.33772	3850.5	4390.8	8.4934	0.30012	3849.9	4390.1	8.4386	0.27004	3849.3	4389.4	8.3895
1000	0.36678	4049.0	4635.8	8.6938	0.32598	4048.4	4635.2	8.6390	0.29333	4047.9	4634.6	8.5900
1100	0.39581	4253.7	4887.0	8.8837	0.35180	4253.2	4886.4	8.8290	0.31659	4252.7	4885.9	8.7800
1200	0.42482	4464.2	5143.9	9.0642	0.37761	4663.7	5143.4	9.0096	0.33984	4463.2	5142.9	8.9606
1300	0.45382	4679.9	5406.0	9.2364	0.40340	4679.4	5405.6	9.1817	0.36306	4679.0	5405.1	9.1382
	P = 2.50 MPa (223.99)				*P* = 3.00 MPa (233.90)				*P* = 3.50 MPa (242.60)			
Sat.	0.07998	2603.1	2803.1	6.2574	0.06668	2604.1	2804.1	6.1869	0.05707	2603.7	2803.4	6.1252
225	0.08027	2605.6	2806.3	6.2638	—	—	—	—	—	—	—	—
250	0.08700	2662.5	2880.1	6.4084	0.07058	2644.0	2855.8	6.2871	0.05873	2623.7	2829.2	6.1748
300	0.09890	2761.6	3008.8	6.6437	0.08114	2750.0	2993.5	6.5389	0.06842	2738.0	2977.5	6.4460
350	0.10976	2851.8	3126.2	6.8402	0.09053	2843.7	3115.3	6.7427	0.07678	2835.3	3104.0	6.6578
400	0.12010	2939.0	3239.3	7.0147	0.09936	2932.7	3230.8	6.9211	0.08453	2926.4	3222.2	6.8404
450	0.13014	3025.4	3350.8	7.1745	0.10787	3020.4	3344.0	7.0833	0.09196	3015.3	3337.2	7.0051
500	0.13998	3112.1	3462.0	7.3233	0.11619	3107.9	3456.5	7.2337	0.09918	3103.7	3450.9	7.1571
600	0.15930	3288.0	3686.2	7.5960	0.13243	3285.0	3682.3	7.5084	0.11324	3282.1	3678.4	7.4338
700	0.17832	3468.8	3914.6	7.8435	0.14838	3466.0	3911.7	7.7571	0.12699	3464.4	3908.8	7.6837
800	0.19716	3655.3	4148.2	8.0720	0.16414	3653.6	4146.0	7.9862	0.14056	3651.8	4143.8	7.9135
900	0.21590	3847.9	4387.6	8.2853	0.17980	3846.5	4385.9	8.1999	0.15402	3845.0	4384.1	8.1275
1000	0.23458	4046.7	4633.1	8.4860	0.19541	4045.4	4631.6	8.4009	0.16743	4044.1	4630.1	8.3288
1100	0.25322	4251.5	4884.6	8.6761	0.21098	4250.3	4883.3	8.5911	0.18080	4249.1	4881.9	8.5191
1200	0.27185	4462.1	5141.7	8.8569	0.22652	4460.9	5140.5	8.7719	0.19415	4459.8	5139.3	8.7000
1300	0.29046	4677.8	5404.0	9.0291	0.24206	4676.6	5402.8	8.9442	0.20749	4675.5	5401.7	8.8723

Sat.	0.04978	2602.3	2801.4	6.0700	0.04406	2600.0	2798.3	6.0198	0.03944	2597.1	2794.3	5.9733
275	0.05457	2667.9	2886.2	6.2284	0.04730	2650.3	2863.1	6.1401	0.04141	2631.2	2838.3	6.0543
300	0.05884	2725.3	2960.7	6.3614	0.05135	2712.0	2943.1	6.2827	0.04532	2697.9	2924.5	6.2083
350	0.06645	2826.6	3092.4	6.5820	0.05840	2817.8	3080.6	6.5130	0.05194	2808.7	3068.4	6.4492
400	0.07341	2919.9	3213.5	6.7689	0.06475	2913.3	3204.7	6.7046	0.05781	2906.6	3195.6	6.6458
450	0.08003	3010.1	3330.2	6.9362	0.07074	3004.9	3323.2	6.8745	0.06330	2999.6	3316.1	6.8185
500	0.08643	3099.5	3445.2	7.0900	0.07651	3095.2	3439.5	7.0300	0.06857	3090.9	3433.8	6.9758
600	0.09885	3279.1	3674.4	7.3688	0.08765	3276.0	3670.5	7.3109	0.07869	3273.0	3666.5	7.2588
700	0.11095	3462.1	3905.9	7.6198	0.09847	3459.9	3903.0	7.5631	0.08849	3457.7	3900.1	7.5122
800	0.12287	3650.1	4141.6	7.8502	0.10911	3648.4	4139.4	7.7942	0.09811	3646.6	4137.2	7.7440
900	0.13469	3843.6	4382.3	8.0647	0.11965	3842.1	4380.6	8.0091	0.10762	3840.7	4378.8	7.9593
1000	0.14645	4042.9	4628.7	8.2661	0.13013	4041.6	4627.2	8.2108	0.11707	4040.3	4625.7	8.1612
1100	0.15817	4248.0	4880.6	8.4566	0.14056	4246.8	4879.3	8.4014	0.12648	4245.6	4878.0	8.3519
1200	0.16987	4458.6	5138.1	8.6376	0.15098	4457.4	5136.9	8.5824	0.13587	4456.3	5135.7	8.5330
1300	0.18156	4674.3	5400.5	8.8099	0.16139	4673.1	5399.4	8.7548	0.14526	4672.0	5398.2	8.7055
		$P = 6.00$ MPa (275.64)				$P = 7.00$ MPa (285.88)				$P = 8.00$ MPa (295.06)		
Sat.	0.03244	2589.7	2784.3	5.8891	0.02737	2580.5	2772.1	5.8132	0.02352	2569.8	2757.9	5.7431
300	0.03616	2667.2	2884.2	6.0673	0.02947	2632.1	2838.4	5.9304	0.02426	2590.9	2785.0	5.7905
350	0.04223	2789.6	3043.0	6.3334	0.03524	2769.3	3016.0	6.2282	0.02995	2747.7	2987.3	6.1300
400	0.04739	2892.8	3177.2	6.5407	0.03993	2878.6	3158.1	6.4477	0.03432	2863.8	3138.3	6.3633
450	0.05214	2988.9	3301.8	6.7192	0.04416	2977.9	3287.0	6.6326	0.03817	2966.7	3272.0	6.5550
500	0.05665	3082.2	3422.1	6.8802	0.04814	3073.3	3410.3	6.7974	0.04175	3064.3	3398.3	6.7239
550	0.06101	3174.6	3540.6	7.0287	0.05195	3167.2	3530.9	6.9486	0.04516	3159.8	3521.0	6.8778
600	0.06525	3266.9	3658.4	7.1676	0.05565	3260.7	3650.3	7.0894	0.04845	3254.4	3642.0	7.0205
700	0.07352	3453.2	3894.3	7.4234	0.06283	3448.6	3888.4	7.3476	0.05481	3444.0	3882.5	7.2812
800	0.08160	3643.1	4132.7	7.6566	0.06981	3639.6	4128.3	7.5822	0.06097	3636.1	4123.8	7.5173

(Continued)

(*Continued*)

T	P = 6.00 MPa (275.64) v	u	h	s	P = 7.00 MPa (285.88) v	u	h	s	P = 8.00 MPa (295.06) v	u	h	s
900	0.08958	3837.8	4375.3	7.8727	0.07669	3835.0	4371.8	7.7991	0.06702	3832.1	4368.3	7.7350
1000	0.09749	4037.8	4622.7	8.0751	0.08350	4035.3	4619.8	8.0020	0.07301	4032.8	4616.9	7.9384
1100	0.10536	4243.3	4875.4	8.2661	0.09027	4240.9	4872.8	8.1933	0.07896	4238.6	4870.3	8.1299
1200	0.11321	4454.0	5133.3	8.4473	0.09703	4451.7	5130.9	8.3747	0.08489	4449.4	5128.5	8.3115
1300	0.12106	4669.6	5396.0	8.6199	0.10377	4667.3	5393.7	8.5472	0.09080	4665.0	5391.5	8.4842
	P = 9.00 MPa (303.40)				P = 10.00 MPa (311.06)				P = 12.50 MPa (327.89)			
Sat.	0.02048	2557.8	2742.1	5.6771	0.01803	2544.4	2724.7	5.6140	0.01350	2505.1	2673.8	5.4623
325	0.02327	2646.5	2855.9	5.8711	0.01986	2610.4	2809.0	5.7568	—	—	—	—
350	0.02580	2724.4	2956.5	6.0361	0.02242	2699.2	2923.4	5.9442	0.01613	2624.6	2826.2	5.7117
400	0.02993	2848.4	3117.8	6.2853	0.02641	2832.4	3096.5	6.2119	0.02000	2789.3	3039.3	6.0416
450	0.03350	2955.1	3256.6	6.4843	0.02975	2943.3	3240.8	6.4189	0.02299	3912.4	3199.8	6.2718
500	0.03677	3055.1	3386.1	6.6575	0.03279	3045.8	3373.6	6.5965	0.02560	3021.7	3341.7	6.4617
550	0.03987	3152.2	3511.0	6.8141	0.03564	3144.5	3500.9	6.7561	0.02801	3124.9	3475.1	6.6289
600	0.04285	3248.1	3633.7	6.9588	0.03837	3241.7	3625.3	6.9028	0.03029	3225.4	3604.0	6.7810
650	0.04574	3343.7	3755.3	7.0943	0.04101	3338.2	3748.3	7.0397	0.03248	3324.4	3730.4	6.9218
700	0.04857	3439.4	3876.5	7.2221	0.04358	3434.7	3870.5	7.1687	0.03460	3422.9	3855.4	7.0536
800	0.05409	3632.5	4119.4	7.4597	0.04859	3629.0	4114.9	7.4077	0.03869	3620.0	4103.7	7.2965
900	0.05950	3829.2	4364.7	7.6782	0.05349	3826.3	4361.2	7.6272	0.04267	3819.1	4352.5	7.5181
1000	0.06485	4030.3	4613.9	7.8821	0.05832	4027.8	4611.0	7.8315	0.04658	4021.6	4603.8	7.7237
1100	0.07016	4236.3	4867.7	8.0739	0.06312	4234.0	4865.1	8.0236	0.05045	4228.2	4858.8	7.9165
1200	0.07544	4447.2	5126.2	8.2556	0.06789	4444.9	5123.8	8.2054	0.05430	4439.3	5118.0	8.0987
1300	0.08072	4662.7	5389.2	8.4283	0.07265	4660.4	5387.0	8.3783	0.05813	4654.8	5381.4	8.2717

Sat.	.010338	2455.4	2610.5	5.3097	.0079204	2390.2	2528.8	5.1418	.0058342	2293.1	2409.7	4.9269
350	.011470	2520.4	2692.4	5.4420	.0017139	1632.0	1662.0	3.7612	.0016640	1612.3	1645.6	3.7280
400	.015649	2740.7	2975.4	5.8810	.0124477	2685.0	2902.8	5.7212	.0099423	2619.2	2818.1	5.5539
450	.018446	2879.5	3156.2	6.1403	.0151740	2844.2	3109.7	6.0182	.0126953	2806.2	3060.1	5.9016
500	.020800	2996.5	3308.5	6.3442	.0173585	2970.3	3274.0	6.2382	.0147683	2942.8	3238.2	6.1400
550	.022927	3104.7	3448.6	6.5198	.0192877	3083.8	3421.4	6.4229	.0165553	3062.3	3393.5	6.3347
600	.024911	3208.6	3582.3	6.6775	.0210640	3191.5	3560.1	6.5866	.0181781	3174.0	3537.6	6.5048
650	.026797	3310.4	3712.3	6.8223	.0227372	3296.0	3693.9	6.7356	.0196929	3281.5	3675.3	6.6582
700	.028612	3410.9	3840.1	6.9572	.0243365	3398.8	3824.7	6.8736	.0211311	3386.5	3809.1	6.7993
800	.032096	3611.0	4092.4	7.2040	.0273849	3601.9	4081.1	7.1245	.0238532	3592.7	4069.8	7.0544
900	.035457	3811.9	4343.8	7.4279	.0303071	3804.7	4335.1	7.3507	.0264463	3797.4	4326.4	7.2830
1000	.038748	4015.4	4596.6	7.6347	.0331580	4009.3	4589.5	7.5588	.0289666	4003.1	4582.5	7.4925
1100	.042001	4222.6	4852.6	7.8282	.0359695	4216.9	4846.4	7.7530	.0314471	4211.3	4840.2	7.6874
1200	.045233	4433.8	5112.3	8.0108	.0387605	4428.3	5106.6	7.9359	.0339071	4422.8	5101.0	7.8706
1300	.048455	4649.1	5375.9	8.1839	.0415417	4643.5	5370.5	8.1093	.0363574	4638.0	5365.1	8.0441
		P = 25 *MPa*				*P* = 30 *MPa*				*P* = 35 *MPa*		
375	.001973	1798.6	1847.9	4.0319	.001789	1737.8	1791.4	3.9303	.001700	1702.9	1762.4	3.8721
400	.006004	2430.1	2580.2	5.1418	.002790	2067.3	2151.0	4.4728	.002100	1914.0	1987.5	4.2124
425	.007882	2609.2	2806.3	5.4722	.005304	2455.1	2614.2	5.1503	.003428	2253.4	2373.4	4.7747
450	.009162	2720.7	2949.7	5.6743	.006735	2619.3	2821.4	5.4423	.004962	2498.7	2672.4	5.1962
500	.011124	2884.3	3162.4	5.9592	.008679	2820.7	3081.0	5.7904	.006927	2751.9	2994.3	5.6281
550	.012724	3017.5	3335.6	6.1764	.010168	2970.3	3275.4	6.0342	.008345	2920.9	3213.0	5.9025
600	.014138	3137.9	3491.4	6.3602	.011446	3100.5	3443.9	6.2330	.009527	3062.0	3395.5	6.1178
650	.015433	3251.6	3637.5	6.5229	.012596	3221.0	3598.9	6.4057	.010575	3189.8	3559.9	6.3010
700	.016647	3361.4	3777.6	6.6707	.013661	3335.8	3745.7	6.5606	.011533	3309.9	3713.5	6.4631
800	.018913	3574.3	4047.1	6.9345	.015623	3555.6	4024.3	6.8332	.013278	3536.8	4001.5	6.7450

(*Continued*)

Table A.1.3
(Continued)

T	v	u	h	s	v	u	h	s	v	u	h	s
	P = 25 MPa				*P = 30 MPa*				*P = 35 MPa*			
900	.021045	3783.0	4309.1	7.1679	.017448	3768.5	4291.9	7.0717	.014883	3754.0	4274.9	6.9886
1000	.023102	3990.9	4568.5	7.3801	.019196	3978.8	4554.7	7.2867	.016410	3966.7	4541.1	7.2063
1100	.025119	4200.2	4828.2	7.5765	.020903	4189.2	4816.3	7.4845	.017895	4178.3	4804.6	7.4056
1200	.027115	4412.0	5089.9	7.7604	.022589	4410.3	5079.0	7.6691	.019360	4390.7	5068.4	7.5910
1300	.029101	4626.9	5354.4	7.9342	.024266	4616.0	5344.0	7.8432	.020815	4605.1	5333.6	7.7652
	P = 40 MPa				*P = 50 MPa*				*P = 60 MPa*			
375	.0016406	1677.1	1742.7	3.8289	.0015593	1638.6	1716.5	3.7638	.0015027	1609.3	1699.5	3.1740
400	.0019077	1854.5	1930.8	4.1134	.0017309	1788.0	1874.6	4.0030	.0016335	1745.3	1843.4	3.9317
425	.0025319	2096.8	2198.1	4.5028	.0020071	1959.6	2060.0	4.2733	.0018165	1892.7	2001.7	4.1625
450	.0036931	2365.1	2512.8	4.9459	.0024862	2159.6	2283.9	4.5883	.0020850	2053.9	2179.0	4.4119
500	.0056225	2678.4	2903.3	5.4699	.0038924	2525.5	2720.1	5.1725	.0029557	2390.5	2567.9	4.9320
600	.0080943	3022.6	3346.4	6.0113	.0061123	2942.0	3247.6	5.8177	.0048345	2861.1	3151.2	5.6451
650	.0090636	3158.0	3520.6	6.2054	.0069657	3093.6	3441.8	6.0342	.0055953	3028.8	3364.6	5.8829
700	.0099415	3283.6	3681.3	6.3750	.0077274	3230.5	3616.9	6.2189	.0062719	3177.3	3553.6	6.0824
800	.0115228	3517.9	3978.8	6.6662	.0090761	3479.8	3933.6	6.5290	.0074588	3441.6	3889.1	6.4110
900	.0129626	3739.4	4257.9	6.9150	.0102831	3710.3	4224.4	6.7882	.0085083	3681.0	4191.5	6.6805
1000	.0143238	3954.6	4527.6	7.1356	.0114113	3930.5	4501.1	7.0146	.0094800	3906.4	4475.2	6.9126
1100	.0156426	4167.4	4793.1	7.3364	.0124966	4145.7	4770.6	7.2183	.0104091	4124.1	4748.6	7.1194
1200	.0169403	4380.1	5057.7	7.5224	.0135606	4359.1	5037.2	7.4058	.0113167	4338.2	5017.2	7.3082
1300	.0182292	4594.3	5323.5	7.6969	.0146159	4572.8	5303.6	7.5807	.0122155	4551.4	5284.3	7.4837

Table A1.4

Compressed Liquid Water

	P = 5.00 MPa (263.99)				P = 10.00 MPa (311.06)				P = 15.00 MPa (342.24)			
T	v	u	h	s	v	u	h	s	v	u	h	s
Sat.	.0012859	1147.78	1154.21	2.9201	.0014524	1393.00	1407.53	3.3595	.0016581	1585.58	1610.45	3.6847
0	.0009977	0.03	5.02	0.0001	.0009952	0.10	10.05	0.0003	.0009928	0.15	15.04	0.0004
20	.0009995	83.64	88.64	0.2955	.0009972	83.35	93.32	0.2945	.0009950	83.05	97.97	0.2934
40	.0010056	166.93	171.95	0.5706	.0010034	166.33	176.36	0.5685	.0010013	165.73	180.75	0.5665
60	.0010149	250.21	255.28	0.8284	.0010127	249.34	259.47	0.8258	.0010105	248.49	263.65	0.8231
80	.0010268	333.69	338.83	1.0719	.0010245	332.56	342.81	1.0687	.0010222	331.46	346.79	1.0655
100	.0010410	417.50	422.71	1.3030	.0010385	416.09	426.48	1.2992	.0010361	414.72	430.26	1.2954
120	.0010576	501.79	507.07	1.5232	.0010549	500.07	510.61	1.5188	.0010522	498.39	514.17	1.5144
140	.0010768	586.74	592.13	1.7342	.0010737	584.67	595.40	1.7291	.0010707	582.64	598.70	1.7241
160	.0010988	672.61	678.10	1.9374	.0010953	670.11	681.07	1.9316	.0010918	667.69	684.07	1.9259
180	.0011240	759.62	765.24	2.1341	.0011199	756.63	767.83	2.1274	.0011159	753.74	770.48	2.1209
200	.0011530	848.08	853.85	2.3254	.0011480	844.49	855.97	2.3178	.0011433	841.04	858.18	2.3103
220	.0011866	938.43	944.36	2.5128	.0011805	934.07	945.88	2.5038	.0011748	929.89	947.52	2.4952
240	.0012264	1031.34	1037.47	2.6978	.0012187	1025.94	1038.13	2.6872	.0012114	1020.82	1038.99	2.6770
260	.0012748	1127.92	1134.30	2.8829	.0012645	1121.03	1133.68	2.8698	.0012550	1114.59	1133.41	2.8575
280					.0013216	1220.90	1234.11	3.0547	.0013084	1212.47	1232.09	3.0392
300					.0013972	1328.34	1342.31	3.2468	.0013770	1316.58	1337.23	3.2259
320									.0014724	1431.05	1453.13	3.4246
340									.0016311	1567.42	1591.88	3.6545

(Continued)

(Continued)

T	P = 20 MPa (365.81) v	u	h	s	P = 30 MPa v	u	h	s	P = 50 MPa v	u	h	s
Sat.	.0020353	1785.47	1826.18	4.0137	—	—	—	—	—	—	—	—
0	.0009904	0.02	20.00	0.0004	.0009856	0.25	29.82	0.0001	.0009766	0.20	49.03	−0.0014
20	.0009928	82.75	102.61	0.2922	.0009886	82.16	111.82	0.2898	.0009804	80.98	130.00	0.2847
40	.0009992	165.15	185.14	0.5646	.0009951	164.01	193.87	0.5606	.0009872	161.84	211.20	0.5526
60	.0010084	247.66	267.82	0.8205	.0010042	246.03	276.16	0.8153	.0009962	242.96	292.77	0.8051
80	.0010199	330.38	350.78	1.0623	.0010156	328.28	358.75	1.0561	.0010073	324.32	374.68	1.0439
100	.0010337	413.37	434.04	1.2917	.0010290	410.76	441.63	1.2844	.0010201	405.86	456.87	1.2703
120	.0010496	496.75	517.74	1.5101	.0010445	493.58	524.91	1.5017	.0010348	487.63	539.37	1.4857
140	.0010678	580.67	602.03	1.7192	.0010621	576.86	608.73	1.7097	.0010515	569.76	622.33	1.6915
160	.0010885	665.34	687.11	1.9203	.0010821	660.81	693.27	1.9085	.0010703	652.39	705.91	1.8890
180	.0011120	750.94	773.18	2.1146	.0011047	745.57	778.71	2.1024	.0010912	735.68	790.24	2.0793
200	.0011387	837.70	860.47	2.3031	.0011302	831.34	865.24	2.2892	.0011146	819.73	875.46	2.2634
220	.0011693	925.89	949.27	2.4869	.0011590	918.32	953.09	2.4710	.0011408	904.67	961.71	2.4419
240	.0012046	1015.94	1040.04	2.6673	.0011920	1006.84	1042.60	2.6489	.0011702	990.69	1049.20	2.6158
260	.0012462	1108.53	1133.45	2.8459	.0012303	1097.38	1134.29	2.8242	.0012034	1078.06	1138.23	2.7860
280	.0012965	1204.69	1230.62	3.0248	.0012755	1190.69	1228.96	2.9985	.0012415	1167.19	1229.26	2.9536
300	.0013596	1306.10	1333.29	3.2071	.0013304	1287.89	1327.80	3.1740	.0012860	1258.66	1322.95	3.1200
320	.0014437	1415.66	1444.53	3.3978	.0013997	1390.64	1432.63	3.3538	.0013388	1353.23	1420.17	3.2867
340	.0015683	1539.64	1571.01	3.6074	.0014919	1501.71	1456.47	3.5425	.0014032	1451.91	1522.07	3.4556
360	.0018226	1702.78	1739.23	3.8770	.0016265	1626.57	1675.36	3.7492	.0014838	1555.97	1630.16	3.6290
380	—	—	—	—	.0018691	1781.35	1837.43	4.0010	.0015883	1667.13	1746.54	3.8100

TABLE A.2 THERMODYNAMIC PROPERTIES OF REFRIGERANT-12*(DICHLORODIFLUOROMETHANE)

Table A.2.1

Saturated Refrigerant-12

Temperature °C t	Pressure MPa p	Specific Volume: Sat. Liquid v_f Cm³/g	Specific Volume: Sat. Vapour v_g m³/kg	Enthalpy: Sat. Liquid h_f	Enthalpy: Evap. h_{fg} kJ/kg	Enthalpy: Sat. Vapour h_g	Entropy: Sat. Liquid s_f kJ/kg K	Entropy: Sat. Vapour s_g
−90	0.0028	0.608	4.415545	−43.243	189.618	146.375	−0.2084	0.8268
−85	0.0042	0.612	3.037316	−38.968	187.608	148.640	−0.1854	0.8116
−80	0.0062	0.617	2.038345	−34.688	185.612	150.924	−0.1630	0.7979
−75	0.0088	0.622	1.537651	−30.401	183.625	153.224	−0.1411	0.7855
−70	0.0123	0.627	1.127280	−26.103	181.640	155.536	−0.1197	0.7744
−65	0.0168	0.632	0.841166	−21.793	179.651	157.857	−0.0987	0.7643
−60	0.0226	0.637	0.637910	−17.469	177.653	160.184	−0.0782	0.7552
−55	0.0300	0.642	0.491000	−13.129	175.641	162.512	−0.0581	0.7470
−50	0.0391	0.648	0.383105	−8.772	173.611	164.840	−0.0384	0.7396
−45	0.0504	0.654	0.302682	−4.396	171.558	167.163	−0.0190	0.7329
−40	0.0642	0.659	0.241910	−0.000	169.479	169.479	−0.0000	0.7269
−35	0.0807	0.666	0.195398	4.416	167.368	171.784	0.0187	0.7214
−30	0.1004	0.672	0.159375	8.854	165.222	174.076	0.0371	0.7165
−25	0.1237	0.679	0.131166	13.315	163.037	176.352	0.0552	0.7121
−20	0.1509	0.685	0.108847	17.800	160.810	178.610	0.0730	0.7082
−15	0.1826	0.693	0.091018	22.312	158.534	180.846	0.0906	0.7046
−10	0.2191	0.700	0.076646	26.851	156.207	183.058	0.1079	0.7014
−5	0.2610	0.708	0.064963	31.420	153.823	185.243	0.1250	0.6986

* Adapted from *Fundamentals of Classical Thermodynamics by*-G. J. Van Wylen and R. Sonntag. John Wiley, New York 1976, P. 667–673 (with the kind permission of the publishers, John Wiley & Sons, Inc, New York).

(Continued)

Table A.2.1
(Continued)

t	p	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_g
0	0.3086	0.716	0.055389	36.022	151.376	187.397	0.1418	0.6960
5	**0.3626**	**0.724**	**0.047485**	**40.659**	**148.859**	**189.518**	**0.1585**	**0.6937**
10	0.4233	0.733	0.040914	45.337	146.265	191.602	0.1750	0.6916
15	**0.4914**	**0.743**	**0.035413**	**50.058**	**143.586**	**193.644**	**0.1914**	**0.6897**
20	0.5673	0.752	0.030780	54.828	140.812	195.641	0.2076	0.6879
25	**0.6516**	**0.763**	**0.026854**	**59.653**	**137.933**	**197.586**	**0.2237**	**0.6863**
30	0.7449	0.774	0.023508	64.539	134.936	199.475	0.2397	0.6848
35	**0.8477**	**0.786**	**0.020641**	**69.494**	**131.805**	**201.299**	**0.2557**	**0.6834**
40	0.9607	0.798	0.018171	74.527	128.525	203.051	0.2716	0.6820
45	**1.0843**	**0.811**	**0.016032**	**79.647**	**125.074**	**204.722**	**0.2875**	**0.6806**
50	1.2193	0.826	0.014170	84.868	121.430	206.298	0.3034	0.6792
55	**1.3663**	**0.841**	**0.012542**	**90.201**	**117.565**	**207.766**	**0.3194**	**0.6777**
60	1.5259	0.858	0.01111	95.665	113.443	209.109	0.3355	0.6760
65	**1.6988**	**0.877**	**0.009847**	**101.279**	**109.024**	**210.303**	**0.3518**	**0.6742**
70	1.8858	0.897	0.008725	107.067	104.255	211.321	0.3683	0.6721
75	**2.0874**	**0.920**	**0.007723**	**113.058**	**99.068**	**212.126**	**0.3851**	**0.6697**
80	2.3046	0.946	0.006821	119.291	93.373	212.665	0.4023	0.6667
85	**2.5380**	**0.976**	**0.006005**	**125.818**	**87.047**	**212.865**	**0.4201**	**0.6631**
90	2.7885	1.012	0.005258	132.708	79.907	212.614	0.4385	0.6585
95	**3.0569**	**1.056**	**0.004563**	**140.068**	**71.658**	**211.726**	**0.4579**	**0.6526**
100	3.3440	1.113	0.003903	148.076	61.768	209.843	0.4788	0.6444
105	**3.6509**	**1.197**	**0.003242**	**157.085**	**49.014**	**206.099**	**0.5023**	**0.6319**
110	3.9784	1.364	0.002462	168.059	28.425	196.484	0.5322	0.6064

Table A.2.3

Superheated Refrigerant-12

t °C	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K	v m/kg	h kJ kg	s kJ kg K
		0.05 MPa			*0.10 MPa*			*0.15 MPa*	
– 20.00	0.341857	181.042	0.7912	0.167701	179.861	0.7401			
– 10.00	0.356227	186.757	0.8133	0.175222	185.707	0.7628	0.114716	184.619	0.7318
00.00	0.370508	192.567	0.8350	0.182647	191.628	0.7849	0.119866	190.660	0.7543
10.00	0.384716	198.471	0.8562	0.189994	197.628	0.8064	0.124932	196.762	0.7763
20.00	0.398863	204.469	0.8770	0.197277	203.707	0.8275	0.129930	202.927	0.7977
30.00	0.412959	210.557	0.8974	0.204506	209.866	0.8482	0.134873	209.160	0.8186
40.00	0.427012	216.733	0.9175	0.211691	216.104	0.8684	0.139768	215.463	0.8390
50.00	0.441030	222.997	0.9372	0.218839	222.421	0.8883	0.144625	221.835	0.8591
60.00	0.455017	229.344	0.9565	0.225955	228.815	0.9078	0.149450	228.277	0.8787
70.00	0.468978	235.774	0.9755	0.233044	235.285	0.9269	0.154247	234.789	0.8980
80.00	0.482917	242.282	0.9942	0.240111	241.829	0.9457	0.159020	241.371	0.9169
90.00	0.496838	248.868	1.0126	0.247159	248.446	0.9642	0.163774	248.020	0.9354
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
00.0	0.088608	189.669	0.7320	0.069752	188.644	0.7139	0.057150	187.583	0.6984
10.00	0.092550	195.878	0.7543	0.073024	194.969	0.7366	0.059984	194.034	0.7216
20.00	0.096418	202.135	0.7760	0.076218	201.322	0.7587	0.062734	200.490	0.7440
30.00	0.100228	208.446	0.7972	0.079350	207.715	0.7801	0.065418	206.969	0.7658
40.00	0.103989	214.814	0.8178	0.082431	214.153	0.8010	0.068049	213.480	0.7869
50.00	0.107710	221.243	0.8381	0.085470	220.642	0.8214	0.070635	220.030	0.8075
60.00	0.111397	227.735	0.8578	0.088474	227.185	0.8413	0.073185	226.627	0.8276
70.00	0.115055	134.291	0.8772	0.091449	233.785	0.8608	0.075750	233.273	0.8473
80.00	0.118690	240.910	0.8962	0.094398	240.443	0.8800	0.078200	239.271	0.8665
90.00	0.122304	247.593	0.9149	0.097327	247.160	0.8987	0.080673	246.723	0.8853

(*Continued*)

Table A.2.2
(*Continued*)

t	v	h	s	v	h	s	v	h	s
100.0	0.125901	254.339	0.9332	0.100238	253.936	0.9171	0.83127	253.530	0.9038
110.0	0.129483	261.147	0.9512	0.103134	260.770	0.9352	0.085566	260.391	0.9220
		0.40 MPa			*0.50 MPa*			*0.60 MPa*	
20.0	0.045836	198.762	0.7199	0.035646	196.935	0.6999			
30.0	0.047971	205.428	0.7423	0.037464	203.814	0.7230	0.030422	202.116	0.7063
40.0	0.050046	212.095	0.7639	0.039214	210.656	0.7452	0.031966	209.154	0.7291
50.0	0.052072	218.779	0.7849	0.040911	217.484	0.7667	0.033450	216.141	0.7511
60.0	0.054059	225.488	0.8054	0.042565	224.315	0.7875	0.034887	223.104	0.7723
70.0	0.056014	232.230	0.8253	0.044184	232.161	0.8077	0.036285	230.062	0.7929
80.0	0.057941	239.012	0.8448	0.045774	238.031	0.8275	0.037653	237.027	0.8129
90.0	0.059846	245.837	0.8638	0.047340	244.932	0.8467	0.038995	244.009	0.8324
100.0	0.061731	252.707	0.8825	0.048886	251.869	0.8656	0.040316	251.016	0.8514
110.0	0.063600	259.624	0.9008	0.050415	258.845	0.8840	0.041619	258.053	0.8700
		0.70 MPa			*0.80 MPa*			*0.90 MPa*	
40.0	0.026761	207.580	0.7148	0.022830	205.924	0.7016	0.019744	204.170	0.6982
50.0	0.028100	214.745	0.7373	0.024068	213.290	0.7248	0.020912	211.765	0.7131
60.0	0.029387	221.854	0.7590	0.025247	220.558	0.7469	0.022012	218.212	0.7358
70.0	0.030632	228.931	0.7799	0.026380	227.766	0.7682	0.023062	226.564	0.7575
80.0	0.031843	235.997	0.8002	0.027477	234.941	0.7888	0.024072	233.856	0.7785
90.0	0.033027	243.066	0.8199	0.028545	242.101	0.8088	0.025051	141.113	0.7987

		0.70 MPa			*0.80 MPa*			*0.90 MPa*	
100.0	**0.034189**	**250.146**	**0.8392**	**0.029588**	**249.260**	**0.8283**	**0.026005**	**248.355**	**0.8184**
110.0	0.035332	257.247	0.8579	0.030612	256.428	0.8472	0.026937	255.593	0.8376
		1.00 MPa			***1.20 MPa***			***1.40 MPa***	
50.0	0.018366	210.162	0.7021	0.014483	206.661	0.6812			
60.0	**0.019410**	**217.810**	**0.7254**	**0.015463**	**214.805**	**0.7060**	**0.012579**	**211.457**	**0.6876**
70.0	0.020397	225.319	0.7476	0.016368	222.687	0.7293	0.013448	219.822	0.7123
80.0	**0.021341**	**232.739**	**0.7689**	**0.017221**	**230.398**	**0.7514**	**0.014247**	**227.891**	**0.7355**
90.0	0.022251	240.101	0.7895	0.018032	237.995	0.7727	0.014997	235.766	0.7575
100.0	**0.023133**	**247.430**	**0.8094**	**0.018812**	**245.518**	**0.7931**	**0.015710**	**243.512**	**0.7785**
110.10	0.023993	254.743	0.8287	0.019567	252.993	0.8129	0.016393	251.170	0.7988
		1.60 MPa			***1.80 MPa***			***2.00 MPa***	
70.0	0.011208	216.650	0.6959	0.009406	213.049	0.6794			
80.0	**0.011984**	**225.177**	**0.7204**	**0.010187**	**222.198**	**0.7057**	**0.008704**	**218.859**	**0.6909**
90.0	0.012698	233.390	0.7433	0.010884	230.835	0.7298	0.009406	228.056	0.7166
100.0	**0.013366**	**241.397**	**0.7651**	**0.011526**	**239.155**	**0.7524**	**0.010035**	**236.760**	**0.7402**
110.0	0.014000	249.264	0.7859	0.012126	2467.264	0.7739	0.010615	245.154	0.7624

TABLE A.3 THERMODYNAMIC PROPERTIES OF REFRIGERANT-22 (MONOCHLORODIFLUOROMETHANE)

Table A.3.1
Saturated Refrigerant-22

		Specific volume (m^3/kg)			Enthalpy (kJ/kg)			Entropy (kJ/kg K)		
Temp. °C	Abs. Press. MPa P	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapour v_g	Sat. Liquid h_f	Evap. h_{fg}	Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
−70	0.0205	0.000670	0.940268	0.94093	−30.607	249.425	218.180	−0.1401	1.2277	1.0876
−65	0.0280	0.000676	0.704796	0.705478	−25.658	246.925	221.267	−0.1161	1.1862	1.0701
−60	0.0375	0.000682	0.536470	0.537152	−20.652	244.354	223.702	−0.0924	1.1463	1.0540
−55	0.0495	0.000689	0.414138	0.414827	−15.585	241.703	226.117	−0.0689	1.1079	1.0390
−50	0.0644	0.000695	0.323862	0.324557	−10.456	238.965	228.509	−0.0457	1.0708	1.0251
−45	0.0827	0.000702	0.256288	0.256990	−5.262	236.132	230.870	−0.0227	1.0349	1.0122
−40	0.1049	0.000709	0.205036	0.205745	0	233.198	233.197	0	1.0002	1.0002
−35	0.1317	0.000717	0.165683	0.166400	5.328	230.156	235.484	0.0225	0.9664	0.9889
−30	0.1635	0.000725	0.135120	0.135844	10.725	227.001	237.726	0.0449	0.9335	0.9784
−25	0.2010	0.000733	0.111126	0.111859	16.191	223.727	239.918	0.0670	0.9015	0.9685
−20	0.2448	0.000741	0.092102	0.092843	21.728	220.327	242.055	0.0890	0.8703	0.9593
−15	0.2957	0.000750	0.076876	0.077625	27.334	216.798	244.132	0.1107	0.8398	0.9505
−10	0.3543	0.000759	0.064581	0.065340	33.012	213.132	246.144	0.1324	0.8099	0.9422
−5	0.4213	0.000768	0.054571	0.055339	38.762	209.323	248.085	0.1538	0.7806	0.9344
0	0.4976	0.000778	0.046357	0.047135	44.586	205.364	249.949	0.1751	0.7518	0.9269
5	0.5838	0.000789	0.039567	0.040356	50.485	201.246	251.731	0.1963	0.7235	0.9197

10	0.6807	0.000800	0.033914	0.034714	56.463	196.960	253.423	0.2173	0.6956	0.9129
15	0.7891	0.000812	0.029176	0.029987	62.523	192.495	255.018	0.2382	0.6680	0.9062
20	0.9099	0.000824	0.025179	0.026003	68.670	187.836	256.506	0.2590	0.6407	0.8997
25	1.0439	0.000838	0.021787	0.022624	74.910	182.968	257.877	0.2797	0.6137	0.8934
30	1.1919	0.000852	0.018890	0.019742	81.250	177.869	259.119	0.3004	0.5867	0.8871
35	1.3548	0.000867	0.016401	0.017269	87.700	172.516	260.216	0.3210	0.5598	0.8809
40	1.5335	0.000884	0.014251	0.015135	94.272	166.877	261.149	0.3417	0.5329	0.8746
45	1.7290	0.000902	0.012382	0.013284	100.982	160.914	261.896	0.3624	0.5058	0.8682
50	1.9423	0.000922	0.010747	0.011669	107.851	154.576	262.428	0.3832	0.4783	0.8615
55	2.1744	0.000944	0.009308	0.010252	114.905	147.800	262.705	0.4042	0.4504	0.8546
60	2.4266	0.000969	0.008032	0.009001	122.180	140.497	262.678	0.4255	0.4217	0.8472
65	2.6999	0.000997	0.006890	0.007887	129.729	132.547	262.276	0.4472	0.3920	0.8391
70	2.9959	0.001030	0.005859	0.006889	137.625	123.772	261.397	0.4695	0.3607	0.8302
75	3.3161	0.001069	0.004914	0.005983	145.986	113.902	259.888	0.4927	0.3272	0.8198
80	3.6623	0.001118	0.004031	0.005149	155.011	102.475	257.486	0.5173	0.2902	0.8075
85	4.0368	0.001183	0.003175	0.004358	165.092	88.598	253.690	0.5445	0.2474	0.7918
90	4.4425	0.001282	0.002282	0.003564	177.204	70.037	247.241	0.5767	0.1929	0.7695
95	4.8835	0.001521	0.001030	0.002551	196.359	34.925	231.284	0.6273	0.0949	0.7222
96.006	4.9773	0.001906	0	0.001906	212.546	0	212.546	0.6708	0	0.6708

Table A.3.2
Superheated Refrigerant-22

Temp. °C	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K
		0.05 MPa			*0.10 MPa*			*0.15 MPa*	
–40	**0.440633**	**234.724**	**1.07616**	**0.216331**	**233.337**	**1.00523**	—	—	—
–30	0.460641	240.602	1.10084	0.226754	239.359	1.03052	0.148723	238.078	0.98773
–20	**0.480543**	**246.586**	**1.12495**	**0.237064**	**245.466**	**1.05513**	**0.155851**	**244.319**	**1.01288**
–10	0.500357	252.676	1.14855	0.247279	251.665	1.07914	0.162879	250.631	1.03733
0	**0.520095**	**258.874**	**1.17166**	**0.257415**	**257.956**	**1.10261**	**0.169823**	**257.022**	**1.06116**
10	0.539771	265.180	1.19433	0.267485	264.345	1.12558	0.176699	263.496	1.08444
20	**0.559393**	**271.594**	**1.21659**	**0.277500**	**270.831**	**1.14809**	**0.183516**	**270.057**	**1.10721**
30	0.578970	278.115	1.23846	0.287467	277.416	1.17017	0.190284	276.709	1.12952
40	**0.598507**	**284.743**	**1.25998**	**0.297394**	**284.101**	**1.19187**	**0.197011**	**283.452**	**1.15140**
50	0.618011	291.478	1.28114	0.307287	290.887	1.21320	0.203702	290.289	1.17289
60	**0.637485**	**298.319**	**1.30199**	**0.317149**	**297.772**	**1.23418**	**0.210362**	**297.220**	**1.19402**
70	0.656935	305.265	1.32253	0.326986	304.757	1.25484	0.216997	304.246	1.21479
80	**0.676362**	**312.314**	**1.34278**	**0.336801**	**311.842**	**1.27519**	**0.223608**	**311.368**	**1.23525**
90	0.695771	319.465	1.36275	0.346596	319.026	1.29524	0.230200	318.584	1.25540
		0.20 MPa			***0.25 MPa***			***0.30 MPa***	
–20	0.115203	243.140	0.98184	—	—	—	—	—	—
–10	**0.120647**	**249.574**	**1.00676**	**0.95280**	**248.492**	**0.98231**	**0.078344**	**247.382**	**0.96170**
0	0.126003	256.069	1.03098	0.099689	255.097	1.00695	0.082128	254.104	0.98677
10	**0.131286**	**262.633**	**1.05458**	**0.104022**	**261.755**	**1.03089**	**0.085832**	**260.861**	**1.01106**
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
20	**0.136509**	**269.273**	**1.07763**	**0.108292**	**268.476**	**1.05421**	**0.089469**	**267.667**	**1.03468**
30	0.141681	275.992	1.10016	0.112508	275.267	1.07699	0.093051	274.531	1.05771

[illegible]	[illegible]	*0.20 MPa*	[illegible]	[illegible]	*0.25 MPa*	[illegible]	[illegible]	*0.30 MPa*	[illegible]
40	0.146809	282.796	1.12224	0.116681	282.132	1.09927	0.096588	281.460	1.08019
50	0.151902	289.686	1.14390	0.120815	289.076	1.12109	0.100085	288.460	1.10220
60	0.156963	296.664	1.16516	0.124918	296.102	1.14250	0.103550	295.535	1.12376
70	0.161997	303.731	1.18607	0.128993	303.212	1.16353	0.106986	302.689	1.14491
80	0.167008	310.890	1.20663	0.133044	310.409	1.18420	0.110399	309.924	1.16569
90	0.171999	318.139	1.22687	0.137075	317.692	1.20454	0.113790	317.241	1.18612
100	0.176972	325.480	1.24681	0.141089	325.063	1.22456	0.117164	324.643	1.20623
110	0.181931	332.912	1.26646	0.145086	332.522	1.24428	0.120522	332.129	1.22603
		0.40 MPa			*0.50 MPa*			*0.60 MPa*	
0	0.060131	252.051	0.95359	—	—	—	—	—	—
10	0.063060	259.023	0.97866	0.049355	257.108	0.95223	0.040180	255.109	0.92945
20	0.065915	266.010	1.00291	0.051751	264.295	0.97717	0.042280	262.517	0.95517
30	0.068710	273.029	1.02646	0.054081	271.483	1.00128	0.044307	269.888	0.97989
40	0.071455	280.092	1.04938	0.056358	278.690	1.02467	0.046276	277.250	1.00378
50	0.074160	287.209	1.07175	0.058590	285.930	1.04743	0.048198	284.622	1.02695
60	0.076830	294.386	1.09362	0.060786	293.215	1.06963	0.050081	292.020	1.04950
70	0.079470	301.630	1.11504	0.062951	300.552	1.09133	0.051931	299.456	1.07149
80	0.082085	308.944	1.13605	0.065090	307.949	1.11257	0.053754	306.938	1.09298
90	0.084679	316.332	1.15668	0.067206	315.410	1.13340	0.055553	314.475	1.11403
100	0.087254	323.796	1.17695	0.069303	322.939	1.15386	0.057332	322.071	1.13466
110	0.089813	331.339	1.19690	0.071384	330.539	1.17395	0.059094	329.731	1.15492
120	0.092358	338.961	1.21654	0.073450	338.213	1.19373	0.060842	337.458	1.17482
130	0.094890	346.664	1.23588	0.075503	345.963	1.21319	0.062576	345.255	1.19441

(Continued)

Table A.3.2
(Continued)

Temp. °C	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K
		0.70 MPa			0.80 MPa			0.90 MPa	
20	0.035487	260.667	0.93565	0.030366	258.737	0.91787	0.026355	256.713	0.90132
30	0.037305	268.240	0.96105	0.032034	266.533	0.94402	0.027915	264.760	0.92831
40	0.039059	275.769	0.98549	0.033632	274.243	0.96905	0.029397	272.670	0.95398
50	0.040763	283.282	1.00910	0.035175	281.907	0.99314	0.030819	280.497	0.97859
60	0.042424	290.800	1.03201	0.036674	289.553	1.01644	0.032193	288.278	1.00230
70	0.044052	298.339	1.05431	0.038136	297.202	1.03906	0.033528	296.042	1.02526
80	0.045650	305.912	1.07606	0.039568	304.868	1.06108	0.034832	303.807	1.04757
90	0.047224	313.527	1.09732	0.040974	312.565	1.08257	0.036108	311.590	1.06930
100	0.048778	321.192	1.11815	0.042359	320.303	1.10359	0.037363	319.401	1.09052
110	0.050313	328.914	1.13856	0.043725	328.087	1.12417	0.038598	327.251	1.11128
120	0.051834	336.696	1.15861	0.045076	335.925	1.14437	0.039817	335.147	1.13162
130	0.053341	344.541	1.17832	0.046413	343.821	1.16420	0.041022	343.094	1.15158
140	0.054836	352.454	1.19770	0.047738	351.778	1.18369	0.042215	351.097	1.17119
150	0.056321	360.435	1.21679	0.049052	359.799	1.20288	0.043398	359.159	1.19047
		1.00 MPa			1.20 MPa			1.40 MPa	
30	0.024600	262.912	0.91358	—	—	—	—	—	—
40	0.025995	271.042	0.93996	0.020851	267.602	0.91411	0.017120	263.861	0.89010
50	0.027323	279.046	0.96512	0.022051	276.011	0.94055	0.018247	272.766	0.91809
60	0.028601	286.973	0.98928	0.023191	284.263	0.96570	0.019299	281.401	0.94441
70	0.029836	294.859	1.01260	0.024282	292.415	0.98981	0.020295	289.858	0.96942
80	0.031038	302.727	1.03520	0.025336	330.508	1.01305	0.021248	298.202	0.99339
90	0.032213	310.599	1.05718	0.026359	308.570	1.03556	0.022167	306.473	1.01649
100	0.033364	318.488	1.07861	0.027357	316.623	1.05744	0.023058	314.703	1.03884

		1.00 MPa			1.20 MPa			1.40 MPa	
110	**0.034495**	**326.405**	**1.09955**	**0.028334**	**324.682**	**1.07875**	**0.023926**	**322.916**	**1.06056**
120	0.035609	334.360	1.12004	0.029292	332.762	1.09957	0.024775	331.128	1.08172
130	**0.036709**	**342.360**	**1.14014**	**0.030236**	**340.871**	**1.11994**	**0.025608**	**339.354**	**1.10238**
140	0.037797	350.410	1.15986	0.031166	349.019	1.13990	0.026426	347.603	1.12259
150	**0.038873**	**358.514**	**1.17924**	**0.032084**	**357.210**	**1.15949**	**0.027233**	**355.885**	**1.14240**
160	0.039940	366.677	1.19831	0.032993	365.450	1.17873	0.028029	364.206	1.16183
		1.60 MPa			1.80 MPa			2.00 MPa	
50	0.015351	269.262	0.89689	0.013052	265.423	0.87625	—	—	—
60	**0.016351**	**278.358**	**0.92461**	**0.014028**	**275.097**	**0.90573**	**0.012135**	**271.563**	**0.88729**
70	0.017284	287.171	0.95068	0.014921	284.331	0.93304	0.013008	281.310	0.91612
80	**0.018167**	**295.797**	**0.97546**	**0.015755**	**293.282**	**0.95876**	**0.013811**	**290.640**	**0.94292**
90	0.019011	304.301	0.99920	0.016546	302.046	0.98323	0.014563	299.697	0.96821
100	**0.019825**	**312.725**	**1.02209**	**0.017303**	**310.683**	**1.00669**	**0.015277**	**308.571**	**0.99232**
110	0.020614	321.103	1.04424	0.018032	319.239	1.02932	0.015960	317.322	1.01546
120	**0.021382**	**329.457**	**1.06576**	**0.018738**	**327.745**	**1.05123**	**0.016619**	**325.991**	**1.03780**
130	0.022133	337.805	1.08673	0.019427	336.224	1.07253	0.017258	334.610	1.05944
140	**0.022869**	**346.162**	**1.10721**	**0.020099**	**344.695**	**1.09329**	**0.017881**	**343.201**	**1.08049**
150	0.023592	354.540	1.12724	0.020759	353.172	1.11356	0.018490	351.783	1.10102
160	**0.024305**	**362.945**	**1.14688**	**0.021407**	**361.666**	**1.13340**	**0.019087**	**360.369**	**1.12107**
170	0.025008	371.386	1.16614	0.022045	370.186	1.15284	0.019673	368.970	1.14070
180	**0.025703**	**379.869**	**1.18507**	**0.022675**	**378.738**	**1.17193**	**0.020251**	**377.595**	**1.15995**
		2.50 MPa			3.00 MPa			3.50 MPa	
70	0.009459	272.677	0.87476	—	—	—	—	—	—
80	0.010243	283.332	0.90537	0.007747	274.530	0.86780	0.005765	262.739	0.82489
90	**0.010948**	**293.338**	**0.93332**	**0.008465**	**286.042**	**0.89995**	**0.006597**	**277.268**	**0.86548**
100	0.011598	302.935	0.95939	0.009098	296.663	0.92881	0.007257	289.504	0.89872
110	**0.012208**	**312.261**	**0.98405**	**0.009674**	**306.744**	**0.95547**	**0.007829**	**300.640**	**0.92818**

(Continued)

Table A.3.2
(Continued)

Temp. °C	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K
		2.50 MPa			3.00 MPa			3.50 MPa	
120	0.012788	321.400	1.00760	0.010211	316.470	0.98053	0.008346	311.129	0.95520
130	0.013343	330.412	1.03023	0.010717	325.955	1.00435	0.008825	321.196	0.98047
140	0.013880	339.336	1.05210	0.011200	335.270	1.02718	0.009276	330.976	1.00445
150	0.014400	348.205	1.07331	0.011665	344.467	1.04918	0.009704	340.554	1.02736
160	0.014907	357.040	1.09395	0.012114	353.584	1.07047	0.010114	349.989	1.04940
170	0.015402	365.860	1.11408	0.012550	362.647	1.09116	0.010510	359.324	1.07071
180	0.015887	374.679	1.13376	0.012976	371.679	1.11131	0.010894	368.590	1.09138
190	0.016364	383.508	1.15303	0.013392	380.695	1.13099	0.011268	377.810	1.11151
200	0.016834	392.354	1.17912	0.013801	389.708	1.15024	0.011634	387.004	1.13115
		4.00 MPa			5.00 MPa			6.00 MPa	
90	0.005037	265.629	0.82544	—	—	—	—	—	—
100	0.005804	280.997	0.86721	0.003334	253.042	0.78005	—	—	—
110	0.006405	293.748	0.90094	0.004255	275.919	0.84064	0.002432	243.278	0.74674
120	0.006924	305.273	0.93064	0.004851	291.362	0.88045	0.003333	272.385	0.82185
130	0.007391	316.080	0.95778	0.005335	304.469	0.91337	0.003899	290.253	0.86675
140	0.007822	326.422	0.98312	0.005757	316.379	0.94256	0.004345	304.757	0.90230
150	0.008226	336.446	1.00710	0.006139	327.563	0.96931	0.004728	317.633	0.93310
160	0.008610	346.246	1.02999	0.006493	338.266	0.99431	0.005071	329.553	0.96094
170	0.008978	355.885	1.05199	0.006826	348.633	1.01797	0.005386	340.849	0.98673
180	0.009332	365.409	1.07324	0.007142	358.760	1.04057	0.005680	351.715	1.01098
190	0.009675	374.853	1.09386	0.007444	368.713	1.06230	0.005958	362.271	1.03402
200	0.010009	384.240	1.11391	0.007735	378.537	1.08328	0.006222	372.602	1.05609
210	0.010335	393.593	1.13347	0.008018	388.268	1.10363	0.006477	382.764	1.07734
220	0.010654	402.925	1.15259	0.008292	397.932	1.12343	0.006722	392.801	1.09790

TABLE A.4 THERMODYNAMIC PROPERTIES OF REFRIGERANT-134A (1, 1, 1, 2-TETRAFLUOROETHANE)

Table A.4.1

Saturated R-134a

Temp. °C	Abs. Press. MPa P	Specific Volume, m³/kg: Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapour v_g	Enthalpy, kJ/kg: Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K: Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
− 33	0.0737	0.000718	0.25574	0.25646	157.417	220.491	377.908	0.8346	0.9181	1.7528
− 30	0.0851	0.000722	0.22330	0.22402	161.118	218.683	379.802	0.8499	0.8994	1.7493
− 26.25	0.1013	0.000728	0.18947	0.19020	165.802	216.360	382.162	0.8690	0.8763	1.7453
− 25	0.0173	0.000730	0.17956	0.18029	167.381	215.569	382.950	0.8754	0.8687	1.7441
− 20	0.1337	0.000738	0.14575	0.14649	173.744	212.340	386.083	0.9007	0.8388	1.7395
− 15	0.1650	0.000746	0.11932	0.12007	180.193	209.004	389.197	0.9258	0.8096	1.7354
− 10	0.2017	0.000755	0.098454	0.099209	186.721	205.564	392.285	0.9507	0.7812	1.7319
− 5	0.2445	0.000764	0.081812	0.082576	193.324	202.016	395.340	0.9755	0.7534	1.7288
0	0.2940	0.000773	0.068420	0.069193	200.000	198.356	398.356	1.0000	0.7262	1.7262
5	0.3509	0.000783	0.057551	0.058334	206.751	194.572	401.323	1.0243	0.6995	1.7239
10	0.4158	0.000794	0.048658	0.049451	213.580	190.652	404.233	1.0485	0.6733	1.7218
15	0.4895	0.000805	0.041326	0.042131	220.492	186.582	407.075	1.0725	0.6475	1.7200
20	0.5728	0.000817	0.035238	0.036055	227.493	182.345	409.838	1.0963	0.6220	1.7183
25	0.6663	0.000829	0.030148	0.030977	234.590	177.920	412.509	1.1201	0.5967	1.7168
30	0.7710	0.000843	0.025865	0.026707	241.790	173.285	415.075	1.1437	0.5716	1.7153
35	0.8876	0.000857	0.022237	0.023094	249.103	168.415	417.518	1.1673	0.5465	1.7139
40	1.0171	0.000873	0.019147	0.020020	256.539	163.282	419.821	1.1909	0.5214	1.7123
45	1.1602	0.000890	0.016499	0.017389	264.110	157.852	421.962	1.2145	0.4962	1.7106

(Continued)